كلية الهندسة – جامعة مصراتة

القسم العام

رياضة "2"

الشيت الثاني

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Integration التكامل

أولاً : التكامل الغير محدود :

هو عملية عكسية للتفاضل . ويكون على الصورة التالية

علامة التكامل ← $\int f(x)\,.dx.$

المتغير المراد التكامل بالنسبة له

الدالة المراد تكاملها

* طرق التكامل :-

① القاعدة الأساسية للتكامل :

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C \quad ; n \neq -1$$

ثابت التكامل .

$$\int k\,dx = kx + c.$$

Q_1: Find the following integrals:

① $\int \frac{3}{x^4}\,dx = \int 3x^{-4}\,dx = -x^{-3} + c = -\frac{1}{x^3} + c.$

② $\int \sqrt{y}\,dy = \int y^{1/2}\,dy = \frac{y^{3/2}}{3/2} + c = \frac{2}{3}\sqrt{y^3} + c.$

③ $\int \frac{5x}{\sqrt[3]{x}}\,dx = \int 5x \,.\, x^{-1/3}\,dx = \int 5x^{2/3}\,dx = 3x^{5/3} + c = 3\sqrt[3]{x^5} + c.$

④ $\int \sqrt{3t^3}\,dt = \int \sqrt{3}\,t^{3/2}\,dx = \frac{2}{5}\sqrt{3}\,t^{5/2} + c = \frac{2}{5}\sqrt{3}\sqrt{t^5} + c$

⑤ $\int \ln 3\,dx = (\ln 3)x + c$

⑥ $\int \frac{1}{e^2}\,dx = \left(\frac{1}{e^2}\right)x + c.$

⑦ $\int 0 \, dx = c.$

⑧ $\int \sin^{-1} 3\pi \, dx = (\sin^{-1} 3\pi) x + c.$

⑨ $\int (r + 3x) \, dx = rx + \frac{3}{2} x^2 + c.$

⑩ $\int (x^3 - 5x + 9) \, dx = \frac{1}{4} x^4 - \frac{5}{2} x^2 + 9x + c.$

⑪ $\int \left(\frac{2}{x^3} - \frac{5}{x^2} + x\right) dx = \int (2x^{-3} - 5x^{-2} + x) \, dx$

$= -x^{-2} + 5x^{-1} + \frac{1}{2} x^2 + C = \frac{-1}{x^2} + \frac{5}{x} + \frac{1}{2} x^2 + C.$

⑫ $\int \left(\frac{x^2 - 25}{x - 5}\right) dx = \int \frac{(\cancel{x-5})(x+5)}{\cancel{x-5}} \, dx = \int (x+5) \, dx$

$= \frac{1}{2} x^2 + 5x + C.$

⑬ $\int \left(\frac{4x^3 - 12x^2}{x - 3}\right) dx = \int \frac{4x^2(\cancel{x-3})}{\cancel{x-3}} \, dx = \int 4x^2 \, dx$

$= \frac{4}{3} x^3 + C.$

⑭ $\int \frac{x \ln 3 + 2 \ln 3}{x + 2} \, dx = \int \frac{(\cancel{x+2}) \ln 3}{\cancel{x+2}} \, dx = \int \ln 3 \, dx$

$= (\ln 3) x + C.$

⑮ $\int t^2 (\sqrt{t^3} + \sqrt[3]{t}) \, dt = \int t^2 (t^{3/2} + t^{1/3}) \, dt = \int (t^{7/2} + t^{7/3}) \, dt$

$= \frac{2}{9} t^{9/2} + \frac{3}{10} t^{10/3} + C = \frac{2}{9} \sqrt{t^9} + \frac{3}{10} \sqrt[3]{t^{10}} + C.$

⑯ $\int \left(x - \frac{1}{x}\right)^2 dx = \int \left(x^2 - 2 + \frac{1}{x^2}\right) dx = \int (x^2 - 2 + x^{-2}) \, dx$

$= \frac{1}{3} x^3 - 2x - \frac{1}{x} + C.$

⑰ $\int x(x-1)^2\,dx = \int x(x^2-2x+1)\,dx = \int (x^3-2x^2-x)\,dx$

$= \frac{1}{4}x^4 - \frac{2}{3}x^3 + \frac{1}{2}x^2 + C.$

⑱ $\int \frac{(x+1)^2}{\sqrt{x}}\,dx = \int \frac{x^2+2x+1}{x^{1/2}}\,dx = \int (x^2+2x+1)\cdot x^{-1/2}\,dx$

$= \int (x^{3/2} + 2x^{1/2} + x^{-1/2})\,dx = \frac{2}{5}x^{5/2} + \frac{4}{3}x^{3/2} + 2x^{1/2} + C.$

$= \frac{2}{5}\sqrt{x^5} + \frac{4}{3}\sqrt{x^3} + 2\sqrt{x} + C.$

② تكامل الدوال المثلثية :

- قاعدة : أي دالة مثلثية نجعلها عبارة عن دالة x تفاضل الزاوية
ثم تكامل الدالة . أي دُ .

$\int \sin(f(x))\, f'(x)\,dx = -\cos(f(x)) + C$

(الدالة) ⟶ (تكاملها)

(الدالة)	(تكاملها)
$\sin x$	$-\cos x$
$\cos x$	$\sin x$
$\sec^2 x$	$\tan x$
$\csc^2 x$	$-\cot x$
$\sec x \tan x$	$\sec x$
$\csc x \cot x$	$-\csc x$
$\tan x$	$\ln\|\sec x\|$
$\cot x$	$\ln\|\sin x\|$
$\sec x$	$\ln\|\sec x + \tan x\|$
$\csc x$	$\ln\|\csc x - \cot x\|$

Q_2: Find the following integrals.

① $\int \sin(3x-2)\,dx. = \boxed{\frac{3}{3}}$ نحتاج الى 3 نضرب الدالة في

تفاضلها → ③

$\frac{1}{3}\int 3.\sin(3x-1)\,dx. = \frac{-1}{3}\cos(3x-1)+C.$

② $\int \sec^2(2y+3)\,dy = \frac{1}{2}\int 2\sec^2(2y+3)\,dy$

② ←

$= \frac{1}{2}\tan(2y+3)+C.$

③ $\int e^x \sin e^x\,dx$

$= -\cos e^x + C.$

ملحوظة

e^x تفاضلها e^x

④ $\int \frac{\cos \ln x}{x}\,dx. = \int \frac{1}{x}.\cos \ln x.\,dx$

$\ln x \xrightarrow{\text{تفاضل}} \frac{1}{x}$

$= \sin \ln x + C.$

⑤ $\int \frac{\cot\sqrt{x}}{\sqrt{x}}\,dx$ ناقص 2 في المقام.

$\sqrt{x} \rightarrow \frac{1}{2\sqrt{x}}$

$2\int \frac{1}{2\sqrt{x}}\cot\sqrt{x}\,dx = 2\ln|\sin\sqrt{x}|+C.$

⑥ $\int \frac{\csc\frac{1}{x}}{x^2}\,dx. = -\int \frac{-1}{x^2}\csc\frac{1}{x}\,dx.$

$+\frac{1}{x} \rightarrow \frac{-1}{x^2}$

$-\ln\left|\csc\frac{1}{x}-\cot\frac{1}{x}\right|+C.$

⑦ $\int \sec^2 y \cdot \tan(\underbrace{\tan y}_{\sec^2 y})\, dy = \ln\left|\sec(\tan y)\right| + c$

⑧ $\int \dfrac{e^x}{\sin^2 e^x}\, dx = \int e^x \cdot \csc^2 e^x\, dx = -\cot e^x + c$

⑨ $\int \dfrac{dx}{x\cos(\ln x)} = \int \dfrac{1}{x} \cdot \sec \ln x\, dx = \ln\left|\sec(\ln x) + \tan(\ln x)\right| + C$

⑩ $\int \dfrac{\tan e^{-3x}}{e^{3x}}\, dx = \int e^{-3x} \tan \underbrace{e^{-3x}}_{\text{تفاضل}}\, dx \rightarrow \boxed{-3}\, e^{-3x}$

$-\frac{1}{3} \int -3 e^{-3x} \tan e^{-3x}\, dx = -\frac{1}{3} \ln\left|\sec e^{-3x}\right| + c$

⑪ $\int (\tan x + \cot x)^2\, dx = \int (\tan^2 x + 2\tan x \cot x + \cot^2 x)\, dx$

$= \int \sec^2 x - \cancel{1} + \cancel{2} \tan x \cdot \dfrac{1}{\tan x} + \csc^2 x \cancel{-1})\, dx$

$\int (\sec^2 x + \csc^2 x)\, dx = \tan x - \cot x + c$

⑫ $\int (\sec x + \tan x)(1 - \sin x)\, dx = \int (\sec x - \underbrace{\sec x \sin x}_{\tan x} + \tan x + \tan x \sin x)\, dx$

$= \int \left(\sec x - \cancel{\tan x} + \cancel{\tan x} - \dfrac{\sin^2 x}{\cos x}\right) dx = \int \left(\sec x - \dfrac{\sin^2 x}{\cos x}\right) dx$

$= \int \left(\sec x - \dfrac{1-\cos^2 x}{\cos x}\right) dx = \int (\cancel{\sec x} - \cancel{\sec x} + \cos x)\, dx = \int \cos x\, dx$

$= \sin x + c$

(13) $\int \frac{1+\cos 2x}{\sin^2 2x}\,dx = \int (\csc^2 2x + \csc 2x \cdot \cot 2x)\,dx$

$= \frac{1}{2}(-\cot 2x - \csc(2x)) + C.$

(14) $\int \frac{\csc\theta}{\csc\theta - \sin\theta}\,d\theta = \int \frac{\frac{1}{\sin\theta}}{\frac{1}{\sin\theta} - \sin\theta}\,d\theta = \int \frac{\frac{1}{\cancel{\sin\theta}}}{\frac{1-\sin^2\theta}{\sin\theta}}\,d\theta.$

$= \int \frac{1}{1-\sin^2\theta}\,d\theta = \int \frac{1}{\cos^2\theta}\,d\theta = \int \sec^2\theta\,d\theta$

$= \tan\theta + C.$

(15) $\int \frac{dx}{\sin x \cos x} = \int \frac{2\,dx}{2\sin x\cos x} = \int 2\csc 2x\,dx$

$= \ln|\csc 2x - \cot 2x| + C.$

(16) $\int \frac{\sin 4x}{\cos 2x}\,dx = \int \frac{2\sin 2x\,\cancel{\cos 2x}}{\cancel{\cos 2x}}\,dx = \int 2\sin 2x\,dx$

$= -\cos 2x + C.$

(17) $\int \frac{dx}{1+\cos x} = \int \frac{dx}{1+\cos x} \cdot \frac{1-\cos x}{1-\cos x} = \int \frac{1-\cos x}{1-\cos x^2}\,dx.$

$\int \frac{1-\cos x}{\sin^2 x}\,dx = \int (\csc^2 x - \csc x \cot x)\,dx$

$= -\cot x + \csc x + C.$

(18) $\int (\sin 3x \cos 2x - \cos 3x \sin 2x)\,dx = \int \sin(3x-2x)\,dx$

$\int \sin x\,dx = -\cos x + C.$

③ تكامل الدوال الأسية : * عشان نقدر نقول أن هذه الدالة أسية لابد أن يكون أساسها ثابت *

قاعدة : أي دالة أسية نجعلها عبارة عن دالة x تفاضل الأس ، ثم نكتب $\frac{\text{نفس الدالة}}{\text{لو الأساس}}$.

أي أن.

$$\int a^{f(x)} \cdot f'(x)\, dx = \frac{a^{f(x)}}{\ln a} + c.$$

$$\int e^{f(x)} \cdot f'(x)\, dx = \frac{e^{f(x)}}{\ln e = 1} + c.$$

$\ln e = 1$

Q3: Find the following integrals.

① $\int x^4\, 7^{x^5+8}\, dx = \frac{1}{5}\int 5x^4\, 7^{x^5+8}\, dx$

$\frac{1}{5} \frac{7^{x^5+8}}{\ln 7} + c.$

② $\int e^x\, 3^{e^x}\, dx = \frac{3^{e^x}}{\ln 3} + c.$

③ $\int \cos x\, 10^{\sin x}\, dx = \frac{10^{\sin x}}{\ln 10} + c.$

④ $\int \frac{x^2}{e^{x^3}}\, dx = \int x^2 e^{-x^3}\, dx = -\frac{1}{3}\int -3x^2 e^{-x^3}\, dx$

$= -\frac{1}{3} e^{-x^3} + c = \frac{-1}{3e^{x^3}} + c.$

⑤ $\int \frac{e^{\tan x}}{\cos^2 x}\, dx = \int \sec^2 x\, e^{\tan x}\, dx = e^{\tan x} + c.$

⑥ $\int \frac{(2^x+1)^2}{2^x}\, dx = \int \frac{2^{2x} + 2^x + 1}{2^x}\, dx = \int 2^x + 2 + 2^{-x}\, dx$

⑦

$$= \frac{2^x}{\ln 2} + 2x - \frac{2^{-x}}{\ln 2} + c,$$

⑦ $$\int \frac{e^{3x}+1}{e^x+1}\,dx = \int \frac{(e^x+1)(e^{2x}-e^x+1)}{(e^x+1)}\,dx$$

$$\int (e^{2x}-e^x+1)\,dx = \frac{1}{2}\int 2e^{2x}\,dx - \int e^x\,dx + \int 1\,dx.$$

$$= \frac{1}{2}e^{2x} - e^x + x + c.$$

⑧ $$\int 2^{3^x}\,3^x\,dx. = \frac{1}{\ln 3}\int 2^{3^x}\,3^x \ln 3\,dx.$$

$3^x \to 3^x \cdot \ln 3$

$$= \frac{1}{\ln 3} \cdot \frac{2^{3^x}}{\ln 2} + c = \frac{2^{3^x}}{\ln 3 \ln 2} + c.$$

⑨ $$\int e^{\ln x + 1}\,dx. = \int e^{\ln x} \cdot e\,dx.$$

$e^{\ln f(x)} = f(x)$

$$e\int x\,dx = \frac{e}{2}x^2 + c.$$

⑩ $$\int e^x \tan^{-1}(3)\,dx = \tan^{-1}(3)\int e^x\,dx = \tan^{-1}(3)\,e^x + c.$$

⑪ $$\int e^{2x^2+\ln x}\,dx = \int e^{2x^2} \cdot e^{\ln x}\,dx = \int e^{2x^2} \cdot x\,dx.$$

$$= \frac{1}{4}\int e^{2x^2}\,4x\,dx = \frac{1}{4}e^{2x^2} + c.$$

⑫ $$\int (\sin 30)^x\,dx = \frac{\sin 30^x}{\ln|\sin 30|} + c.$$

⑬ $\int (2+e)^x \, dx = \dfrac{(2+e)^x}{\ln(2+e)} + c.$

⑭ $\int \dfrac{2^{\log x}}{x} \, dx. = \ln 10 \int \dfrac{2^{\log x}}{x \ln 10} dx = \dfrac{2^{\log x} \ln 10}{\ln 2} + c.$

⑮ $\int \log\left(\dfrac{2^{x+1}}{3^x}\right) dx = \int (\log 2^{x+1} - \log 3^x) \, dx$

$\log a^n = n \log a$

$\int ((x+1)\log 2 - \cancel{x} \log 3) \, dx = (\frac{1}{2}x^2 + x) \log 2 - \frac{1}{2}x^2 \log 3 + c.$

④ تكامل الدوال الزائدية :

قاعدة : اي دالة زائدية نجعلها عبارة عن دالة x تفاضل الزاوية ثم تكامل الدالة

اي أن :

~~$\int \sinh(f(x)) \, dx$~~

$\sinh x = \dfrac{e^x - e^{-x}}{2}.$

$\int \sinh(f(x)) \cdot f'(x) \, dx = \cosh(f(x)) + c.$

$\cosh x = \dfrac{e^x + e^{-x}}{2}$

الدالة	تكاملها
$\sinh(x)$	$\cosh x$
$\cosh(x).$	$\sinh x$
$\operatorname{sech}^2(x).$	$\tanh x$
$\operatorname{csch}^2 x$	$-\coth x$
$\operatorname{sech} x \tanh x.$	$-\operatorname{sech} x.$
$\operatorname{csch} x \coth x.$	$-\operatorname{csch} x.$
$\tanh x.$	$\ln\lvert\cosh x\rvert$
$\coth x.$	$\ln\lvert\sinh x\rvert$
$\operatorname{sech} x.$	~~$\tan^{-1}(\tanh \frac{x}{2})$~~ $\tan^{-1}(\sinh x)$
$\operatorname{csch} x.$	$\ln\lvert\tanh \frac{x}{2}\rvert$

Q4: Find the following integrals.

① $\int \sinh(2x+3)\,dx = \frac{1}{2}\int 2\sinh(2x+3)\,dx = \cosh(2x+3)+C.$

② $\int e^x \operatorname{sech}^2 e^x\,dx = \tanh e^x + C.$

③ $\int \frac{x}{\operatorname{sech} x^2}\,dx = \int x \cdot \cosh x^2\,dx = \frac{1}{2}\int 2x\cosh x^2\,dx.$

$= \frac{1}{2}\sinh x^2 + C$

④ $\int \frac{dx}{x\operatorname{csch}(\ln x)} = \int \frac{1}{x}\sinh(\ln x)\,dx = \cosh(\ln x) + C.$

⑤ $\int \cosh x\, e^{\sinh x}\,dx = e^{\sinh x} + C.$

⑥ $\int \frac{4}{e^{2x}+e^{-2x}+2}\,dx = \int \left(\frac{2}{e^x+e^{-x}}\right)^2 dx.$

$\int \operatorname{sech}^2 x\,dx = \tanh x + C.$

⑦ $\int \frac{\cosh\theta}{\sinh\theta+\cosh\theta}\,d\theta = \int \frac{\frac{1}{2}(e^\theta+e^{-\theta})}{\frac{1}{2}(e^\theta-e^{-\theta})+\frac{1}{2}(e^\theta+e^{-\theta})}\,d\theta.$

$\int \frac{\frac{1}{2}(e^\theta+e^{-\theta})}{e^\theta}\,d\theta = \int \frac{1}{2}(1+e^{-2\theta})\,d\theta$

$\frac{1}{2}(\theta - \frac{1}{2}e^{-2\theta}) + C.$

⑤ تكامل القوس :

قاعدة : لتكامل أي دالة قوس مرفوعة لأس (n). نجعلها عبارة عن
القوس x تفاضل ما بداخل القوس . ثم تكامل القوس .

$$\int (f(x))^n f'(x)\, dx = \frac{(f(x))^{n+1}}{n+1} + c, \qquad n \neq -1$$

Q5: Find the following int.

① $\int t^{2/3} (\underline{t^{5/3}} + 1)^{2/3}\, dt.$ $\xrightarrow{\text{تفاضل}}$ $\frac{5}{3} t^{2/3}.$ نحتاج لـ $\frac{5}{3}$

$$\frac{3}{5}\int \frac{5}{3} t^{2/3} (t^{5/3}+1)^{2/3}\, dt = \frac{3}{5}(t^{5/3}+1)^{5/3} \Big/ \tfrac{5}{3} + c.$$

$$= \frac{9}{25}(t^{5/3}+1)^{5/3} + c$$

② $\int e^x (6+e^x)^5\, dx = (6+e^x)^6 + c.$

③ $\int \frac{\ln x}{x}\, dx = \int \frac{1}{x} \cdot \ln x\, dx = \frac{(\ln x)^2}{2} + c.$

④ $\int \frac{(4+\sqrt{x})^4}{\sqrt{x}}\, dx = 2\int \frac{1}{2\sqrt{x}} (4+\sqrt{x})^4\, dx$

$$= \frac{2}{5}(4+\sqrt{x})^5 + c.$$

⑤ $\int \frac{(5+\frac{1}{x})^5}{x^2}\, dx = -\int \frac{-1}{x^2} (5+\frac{1}{x})^5\, dx = -\frac{1}{6}(5+\frac{1}{x})^6 + c.$

⑥ $\int \frac{1}{x^2} \sin\frac{1}{x} \cos\frac{1}{x}\, dx.$

$\int \frac{1}{x^2} \cos\frac{1}{x} (\sin\frac{1}{x})' \, dx = -\int \frac{-1}{x^2} \cos\frac{1}{x} (\sin\frac{1}{x})' \, dx$

$= -\frac{1}{2} (\sin\frac{1}{x})^2 + c.$

⑦ $\int \sec^7 2x \cdot \tan 2x\, dx.$

$= \frac{1}{2} \int 2 \sec 2x \tan 2x\, (\sec 2x)^6\, dx.$

$= \frac{1}{14} (\sec 2x)^7 + c.$

⑧ $\int \sinh^{11}(2x) \cdot \cosh(2x)\, dx.$

$= \frac{1}{2} \int (\sinh)^{11}(2x) \cdot \cosh(2x)\, dx = \frac{1}{24} \sinh^{12}(2x) + c.$

⑨ $\int \frac{x+1}{\sqrt{x^2+2x-3}}\, dx = \int (x+1)(x^2+2x-3)^{-1/2}\, dx.$

$\frac{1}{2} \int 2(x+1)(x^2+2x-3)^{-1/2}\, dx = (x^2+2x-3)^{1/2} + c.$

⑩ $\int \frac{dx}{x^{1/5}\sqrt{1+x^{4/5}}} = \frac{5}{4} \int \frac{4}{5} x^{-1/5} \cdot (1+x^{4/5})^{-1/2}\, dx.$

$= \frac{5}{2} (1+x^{4/5})^{1/2} + c.$

(11) $\int \frac{\operatorname{csch}^2 x}{\sqrt[3]{\coth x}}\, dx = -\int -\operatorname{csch}^2 x\, (\coth x)^{-1/3}\, dx.$

$= \frac{-3}{2} (\coth x)^{2/3} + c.$

(12) $\int \frac{\cos^2 x}{\csc x}\, dx = \int \sin x \cdot (\cos x)^2 \cdot dx.$

$= -\int -\sin x\, (\cos x)^2 dx = \frac{-\cos^3 x}{3} + c.$

(13) $\int \sin t \cos t\, (\sin t + \cos t)\, dt.$

$= \int \left[(\sin t)^2 \cdot \cos t + \sin t\, (\cos t)^2\right] dt = \frac{1}{3} \sin^3 t - \frac{1}{3} \cos^3 t + c.$

$\frac{1}{3}(\sin^3 t - \cos^3 t) + c.$

(14) $\int \sqrt{1+\sin y}\, dy. = \int \sqrt{1+\sin y} \cdot \frac{\sqrt{1-\sin y}}{\sqrt{1-\sin y}}\, dx$

$\int \frac{\sqrt{1-\sin^2 y}}{\sqrt{1-\sin y}} \cdot dy = \int \frac{\cos y}{\sqrt{1-\sin y}}\, dy$

$= -\int -\cos y\, (1-\sin y)^{-1/2}\, dy = -2 (1-\sin y)^{1/2} + c$

$= -2\sqrt{1-\sin y} + c.$

(13)

(6) نتيجة (التكامل باستخدام اللوغاريتم الطبيعي):
قاعدة: إذا كان البسط عبارة عن تفاضل المقام فإن التكامل لوغاريتم المقام،
أي أن:

$$\int \frac{f'(x)}{f(x)}\,dx = \ln|f(x)| + c.$$

Q_6: Find the following int.

(1) $\displaystyle\int \frac{e^x}{2+e^x}\,dx = \ln|2+e^x| + c.$

(2) $\displaystyle\int \frac{dx}{x^2(1+\frac{1}{x})} = \int \frac{\frac{1}{x^2}}{(1+\frac{1}{x})}\,dx = -\int \frac{-1/x^2}{(1+1/x)}\,dx$

$\displaystyle = -\ln(1+\tfrac{1}{x}) + c.$

(3) $\displaystyle\int \frac{\sec^2 x}{2+\tan x}\,dx = \ln|2+\tan x| + c$

(4) $\displaystyle\int \frac{\cosh 3x}{1+\sinh 3x}\,dx = \frac{1}{3}\int \frac{3\cosh 3x}{1+\sinh 3x}\,dx.$

$\displaystyle = \frac{1}{3}\ln|1+\sinh 3x| + c.$

(5) $\displaystyle\int \frac{dx}{(x^2+1)(2+\tan^{-1}x)}\,dx = \ln|2+\tan^{-1}x| + c.$

$\displaystyle \tan^{-1}x = \frac{1}{x^2+1}$ $\longrightarrow$ تفاضل الدوال المثلثية العكسية

تذكير!!

⑥ $\int \frac{\sqrt{x}}{1+x\sqrt{x}}\, dx = \int \frac{x^{1/2}}{1+x^{3/2}}\, dx = \frac{2}{3}\int \frac{\frac{3}{2}\, x^{1/2}}{1+x^{3/2}}\, dx$

$= \frac{2}{3}\ln\left|1+x^{3/2}\right| + c.$

⑦ $\int \frac{e^{x}-e^{-x}}{e^{x}+e^{-x}}\, dx = \ln\left|e^{x}+e^{-x}\right| + c$

$\boxed{\begin{array}{l} e^{x} \longrightarrow e^{x} \\ e^{-x} \longrightarrow -e^{-x} \end{array}}$ تذكير!!

⑧ $\int \frac{\cot x}{\ln(e\sin x)}\, dx = \ln\left|\ln(e\sin x)\right| + c.$

⑨ $\int \frac{x^{2}+2x+2}{x+2}\, dx = \int \left(x+\frac{2}{x+2}\right) dx$

$= \frac{1}{2}x^{2} + \ln|x+2$

$$\begin{array}{r|l} & x + \frac{2}{x+2} \\ \hline x+2 & x^{2}+2x+2 \\ & x^{2}+2x \\ \hline & 2 \end{array}$$

الحد العلوي

$$\int_a^b f(x)\,dx$$

الحد السفلي

* ثانياً: التكامل المحدود:-

* خواص التكامل المحدود:

① $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx.$

② $\int_a^a f(x)\,dx = 0$

③ $\int_a^b f(x)\,dx + \int_b^c f(x)\,dx = \int_a^c f(x)\,dx.$

④ $\int_a^b [c_1 f(x) \pm c_2 g(x)]\,dx = \int_a^b c_1 f(x)\,dx \pm \int_a^b c_2 g(x)\,dx.$

⑤ if $f(x)$ even (زوجية) $\longrightarrow \int_{-a}^{a} f(x)\,dx = 2\int_0^a f(x)\,dx.$

⑥ if $f(x)$ odd (فردية) $\longrightarrow \int_{-a}^{a} f(x)\,dx = 0.$

ex:

① $\int_0^2 (x^2-2x)\,dx = \frac{1}{3}x^3 - x^2\Big|_0^2 = \frac{8}{3} - 4 = \boxed{\frac{-4}{3}}$

② $\int_\pi^e e^e\,dx = e^e x\Big|_\pi^e = \boxed{e^e(e-\pi)}$

(3) $\displaystyle\int_0^3 |3-x^2|\,dx.$ $\qquad 3-x^2=0 \to x^2=3$

$x=\pm\sqrt{3}$

$$=\int_0^{\sqrt{3}}(3-x^2)\,dx+\int_{\sqrt{3}}^{3}-(3-x^2)\,dx.$$

$$I = 3x-\frac{x^3}{3}\Bigg|_0^{\sqrt{3}} - 3x+\frac{x^3}{3}\Bigg|_{\sqrt{3}}^{3}.$$

$$=(3\sqrt{3}-\sqrt{3})-\cancel{(9-9)}+(3\sqrt{3}-\sqrt{3}) = \boxed{4\sqrt{3}}$$

(4) $\displaystyle\int_{-1}^{-1}(x^2+3x+5)^{99}\,dx \ \boxed{=0} \to$ (odd)

(5) $\displaystyle\int_{-\pi/2}^{\pi/2}\sqrt{1-\cos^2 x}.\,dx = \int_{-\pi/2}^{\pi/2}\sqrt{\sin^2 x}.\,dx.$

$$\int_{-\pi/2}^{\pi/2}|\sin^2 x|\,dx =$$

$$\int_{-\pi/2}^{0}-\sin x\,dx+\int_{0}^{\pi/2}\sin x\,dx.$$

$$I=\cos x\Big|_{-\pi/2}^{0}+(-\sin x)\Big|_{0}^{\pi/2} = (1-0)-(0-1)$$

$$1+1 = \boxed{2}$$

⑥ $\displaystyle\int_1^2 \frac{x-1}{x^2-2x+1}\,dx = \frac{1}{2}\int_1^2 \frac{2(x-1)}{x^2-2x+2}\,dx.$

$$I = \frac{1}{2}\ln|x^2-2x+2|\Big|_1^2 = \frac{1}{2}(\ln_2 - \ln_1) \quad \boxed{= \frac{1}{2}\ln|2|}$$

⑦ $\displaystyle\int_e^{e^2} \frac{dx}{x\ln x} = \int_e^{e^2} \frac{1/x}{\ln x}\,dx.$

$$I = \ln|\ln|x||\Big|_e^{e^2} = \ln|\ln e^2| - \ln|\ln e|$$

$$\ln|2\ln e| - 0 \quad \boxed{= \ln 2}$$

⑧ $\displaystyle\int_0^{\ln 2} \frac{dw}{1+e^w} = \int_0^{\ln 2} \frac{1}{1+e^w}\,dw.$

$$\int_0^{\ln 2} \frac{1}{1+e^w}\cdot\frac{e^{-w}}{e^{-w}}\,dw = \int_0^{\ln 2} \frac{e^{-w}}{e^{-w}+1}\,dw.$$

$$-\int_0^{\ln 2} \frac{-e^{-w}}{e^{-w}+1}\,dw. = -\ln|e^{-w}+1|\Big|_0^{\ln 2}.$$

$$I = -\left[\ln\frac{3}{2} - \ln 2\right] = \ln 2 - \ln\frac{3}{2} = \ln\frac{2}{3/2} \quad \boxed{= \ln 4/3}$$

⑨ $\int_{-1}^{6} |x^2-4|\,dx \rightsquigarrow |x^2-4|=0 \to x^2-4=0 \to x=\pm 2.$

$= \int_{-1}^{2} -(x^2-4)\,dx + \int_{2}^{6} (x^2-4)\,dx.$

$= \frac{1}{3}x^3+4x\Big|_{-1}^{2} + \frac{1}{3}x^3 - 4x\Big|_{2}^{6} = \left(\frac{-8}{3}+8\right)-\left(\frac{1}{3}-4\right)+72-24-\left(\frac{8}{3}-8\right).$

$I = 68 - \frac{17}{3} = \boxed{\frac{187}{3}}$

⑩ $\int_{-2}^{0} |9-x^2|\,dx. \rightsquigarrow |9-x^2|=0 \to x=\pm 3.$

$\int_{-2}^{0}(9-x^2)\,dx. = 9x-\frac{1}{3}x^3\Big|_{-2}^{0}$

$I = 0-\left(-18+\frac{8}{3}\right) = 18-\frac{8}{3} = \boxed{\frac{46}{3}}$

⑪ $\int_{1}^{2} \frac{(2x+3)^{-1}}{\log 3}\,dx = \frac{1}{\log_3}\int_{1}^{2} \frac{1}{2x+3}\,dx.$

$\frac{1}{2\log 3}\int_{1}^{2}\frac{2}{2x+3}\,dx = \frac{1}{2\log_3}\ln|2x+3|\Big|_{1}^{2}.$

$I = \frac{1}{2\log_3}\left[\ln 7-\ln 5\right] = \boxed{\frac{1}{2\log_3}\ln\left(\frac{7}{5}\right)}$

⑫