ENGINEERING

كلية الهندسة – جامعة مصراتة

القسم العام

رياضة "2"

الشيت الأول

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Complex Numbers

لو أخذنا المعادلة التربيعية على الصورة العامة $ax^2+bx+c=0$

فيكون نتيجة ايجاد جذورها باستخدام المميز

$$x_{1,2}=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

فهذه المعادلة نتائجها أعداد طبيعية اذا كان

المقدار $b^2-4ac>0$

ولكن !

اذا كان $b^2-4ac<0$ فهنا يظهر لنا عدد سالب تحت الجذر

فلا نستطيع ايجاد جذر بقيمة سالبة فيظهر لنا الأعداد المركبة.

مثال: $x^2-x-2=0 \longrightarrow a=1,\ b=-1,\ c=-2$

$$x_{1,2}=\frac{-(-1)\pm\sqrt{(-1)^2-(4\times1\times-2)}}{2}$$

$$x_1=2,\quad x_2=-1$$

ومثال: $x^2+16=0 \rightarrow x^2=-16$

$x=\sqrt{-16}$ ← عدد مركب (تخيلي)

$$i=\sqrt{-1},\quad i^2=\sqrt{-1}\times\sqrt{-1}=-1$$

$$\sqrt{-5}=\sqrt{(-1)(+5)}=\sqrt{-1}\,\sqrt{+5}=\sqrt{5}\,i$$

$$\sqrt{-16}=\sqrt{-1}\,\sqrt{16}=\sqrt{16}\,i=4i$$

$$\sqrt{-9}=\sqrt{-1}\,\sqrt{9}=\sqrt{9}\,i=3i$$

وهكذا ....

$$Z = x + iy \qquad x, y \in \mathbb{R}.$$

$Re(z)$. الجزء الحقيقي Real part

$Im(z)$. الجزء التخيلي Img. part

Complex Numbers.

$\bar{Z} = x - iy$ ← مرافق العدد التخيلي

* الصورة القياسية للعدد المركب

$Z = x + iy$ ← الصورة القياسية standard Form

العمليات على الأعداد المركبة :

① الجمع والطرح : نجمع او نطرح الجزء الحقيقي مع الجزء الحقيقي في العدد الثاني والتخيلي مع التخيلي

② الضرب : نضرب ضرب عادي جداً أي نفك المقدار مراعاة (i)

③ القسمة : نقسم عادي لأي رقم حقيقي ولكن مع مراعاة عدم وجود جزء تخيلي في المقام.

example:

$$Z_1 = 1 + 3i \quad , \quad Z_2 = 5 - 7i \quad , \quad Z_3 = -8i$$

احسب :

$$Z_1 + Z_2 \quad , \quad Z_2 - Z_1 \quad , \quad \frac{Z_1}{Z_2} \quad , \quad \frac{Z_1}{Z_3} \quad , \quad Z_1 Z_3 \quad , \quad Z_2 Z_1$$

① $z_1 + z_2 =$

$$(1 + 3i) + (5 - 7i).$$

$$6 - 4i.$$

② $z_2 - z_1 =$

$$(5 - 7i) + (1 + 3i)$$

$$4 - 10i$$

③ $z_1 z_3 =$

$$(1 + 3i)(-8i).$$

$$-8i + 24.$$

④ $z_2 \cdot z_1 =$

$$(5 - 7i)(1 + 3i)$$

$$5 + 15i - 7i + 21.$$

$$26 + 8i$$

$Re(z_2 z_1) = 26.$

$Im(z_2 z_1) = 8.$

⑤ $\dfrac{z_1}{z_2} = \dfrac{1 + 3i}{5 - 7i} \cdot \dfrac{5 + 7i}{5 + 7i} = \dfrac{5 + 7i + 15i - 21}{25 + \cancel{35}i - \cancel{35}i + 49}$

$$= \frac{-16 + 22i}{74} = \frac{-16}{74} + \frac{22}{74}i = \boxed{\frac{-8}{37} + \frac{11}{37}i}$$ standard form

⑥ $\dfrac{z_1}{z_3} = \dfrac{1 + 3i}{-8i} \cdot \dfrac{8i}{8i} = \dfrac{8i - 24}{64}$

$$\frac{-24}{64} + \frac{8i}{64} = \boxed{\frac{-3}{8} + \frac{1}{8}i}$$ stan form.

③

modulus of z مقياس العدد المركب

$$|z| = \sqrt{a^2 + b^2}$$

example prove that $z \cdot \bar{z} = |z|^2$.

let $z = a + ib$, $\bar{z} = a - ib$

$$z\bar{z} = (a + bi)(a - bi)$$

$$a^2 + \cancel{iab} + b^2 - \cancel{iab} = \underline{a^2 + b^2} = \underline{|z|^2}$$

prove that

① $Re(z) = \dfrac{z + \bar{z}}{2}$

$$\frac{z + \bar{z}}{2} = \frac{(a + i\cancel{b}) + (a - i\cancel{b})}{2} = \frac{2a}{2} = a$$

$$\boxed{Re(z) = a}$$

② $Im(z) = \dfrac{z - \bar{z}}{2i}$

$$\frac{z - \bar{z}}{2i} = \frac{(\cancel{a} + ib) - (\cancel{a} - ib)}{2i} = \frac{\cancel{2i}\, b}{\cancel{2i}} = b$$

$$\boxed{Im(z) = b}$$

The power of (i). القوى للجذر التخيلي

$i = \sqrt{-1}$.

$i^2 = -1$

$i^3 = i^2 \cdot i = -i$

$i^4 = i^3 \cdot i = 1$

$i^5 = i$

$i^6 = -1$

$i^7 = -i$

$i^8 = 1$

$i^9 = i$

$i^{10} = -1$

وهكذا

$i^{4n} = 1$

When $n = 1, 2, 3 ----$

$i^{2n} = -1$

When (n) عدد فردي

The square root of a Complex Numbers.

ex: Find

① $\sqrt{1+i} = u + iv$

$1 + i = (u + iv)^2$. $\rightarrow$ $1 + i = u^2 + 2uvi - v^2$.

$1 = u^2 - v^2 \rightarrow$ ①

$1 = 2uv \rightarrow$ ⊛ $\qquad v = \frac{1}{2u} \rightarrow$ ②

نعوض في ①

$$1 = u^2 - \frac{1}{4u^2} \quad \times u^2$$

$$u^2 = u^4 - \frac{1}{4} \longrightarrow u^4 - u^2 - \frac{1}{4} = 0$$

نوضع $u^2 = x$

$$x^2 - x - 1/4 = 0$$

$$a = 1 \;,\; b = -1 \;,\; c = -1/4$$

$$x_{1,2} = \frac{1 \pm \sqrt{2}}{2} \longrightarrow u^2 = \frac{1+\sqrt{2}}{2}$$

نرجع نعوض بـ (u^2) في ①

$$v^2 = \frac{1+\sqrt{2}}{2} - 1$$

$$v^2 = \frac{1+\sqrt{2}-2}{2} \longrightarrow v^2 = \frac{\sqrt{2}-1}{2}$$

$$u = \sqrt{\frac{1+\sqrt{2}}{2}} \;,\; v = \sqrt{\frac{\sqrt{2}-1}{2}}$$

$$\sqrt{1+i} = \sqrt{\frac{1+\sqrt{2}}{2}} + i\sqrt{\frac{\sqrt{2}-1}{2}} \quad *$$

② $\sqrt{1+2i} = x + yi$

$$1+2i = (x+yi)^2 \longrightarrow 1 + 2i = x^2 + 2xyi - y^2$$

$$1 = x^2 - y^2 \longrightarrow ①$$

$$\cancel{2} = \cancel{2}xy \longrightarrow y = \frac{1}{x} \longrightarrow ②$$

$$1 = x^2 - \frac{1}{x^2} \leftarrow \cdot x^2$$ نعوض في ①

$$x^2 = x^4 - 1$$

$$x^4 - x^2 - 1 = 0.$$

$\boxed{\text{بوضع } u = x^2}$

$$u^2 - u - 1 = 0$$

$$u = \frac{1 \pm \sqrt{5}}{2}, \quad x^2 = \frac{1 + \sqrt{5}}{2}.$$

نرجع نعوض بـ (x^2) في ①

$$1 = \frac{1 + \sqrt{5}}{2} - y^2$$

$$y^2 = \frac{1 + \sqrt{5}}{2} - 1 \longrightarrow y^2 = \frac{1 + \sqrt{5} - 2}{2}.$$

$$y^2 = \frac{\sqrt{5} - 1}{2}.$$

$$x = \sqrt{\frac{1 + \sqrt{5}}{2}}, \quad y = \sqrt{\frac{\sqrt{5} - 1}{2}}$$

$$\sqrt{1 + 2i} = \sqrt{\frac{1 + \sqrt{5}}{2}} + i\sqrt{\frac{\sqrt{5} - 1}{2}}$$

polar form of Complex Numbers.

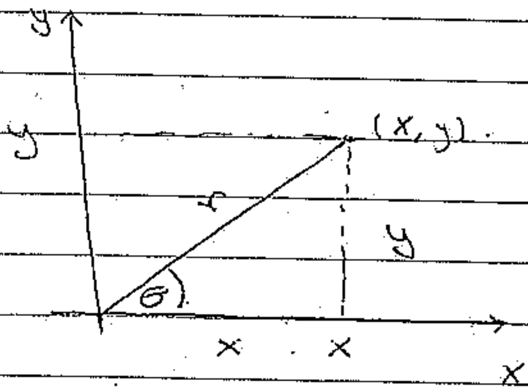
(الأعداد المركبة في الصورة القطبية)

$Z = x + iy$ stan. form.

$x, y \in R$

$Z = r\cos\theta + ri\sin\theta$.

$Z = r(\cos\theta + i\sin\theta)$.



$Z = re^{i\theta}$

Euler formula صيغة أويلر

$e^{i\theta} = \cos\theta + i\sin\theta$.

$\cos\theta = \frac{x}{r}$.

$r^2 = x^2 + y^2$

$x = r\cos\theta \rightarrow (*)$

$|Z| = r = \sqrt{x^2 + y^2}$

$\sin\theta = \frac{y}{r}$.

$\theta = \tan^{-1}\left(\frac{y}{x}\right) \rightarrow Arg(z) = \theta$

$y = r\sin\theta \rightarrow (**)$

$r^2 = x^2 + y^2$

$y = r\sin\theta$

$r^2\cos^2\theta + r^2\sin^2\theta$

$x = r\cos\theta$

$r^2(\cos^2\theta + \sin^2\theta)$

$r^2(1) = r^2$.

$\frac{y}{x} = \frac{\sin\theta}{\cos\theta}$

$\tan\theta = \frac{y}{x} \rightarrow \theta = \tan^{-1}\left(\frac{y}{x}\right)$

$x = r\cos\theta$ التحويل من الصورة

$y = r\sin\theta$ القطبية إلى القياسية.

$|z| = r = \sqrt{x^2+y^2}$ التحويل من الصورة

$\theta = \tan^{-1}\left(\frac{y}{x}\right)$. القياسية إلى القطبية.

example: Write in standard form.

① $4\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$. $\rightarrow r = 4, \ \theta = \frac{\pi}{6} = 30^\circ$

$x = r\cos\theta = 4\cdot\cos\frac{\pi}{6} = 2\sqrt{3}$.

$y = r\sin\theta = 4\sin\frac{\pi}{6} = 2$.

stan. form $\rightarrow 2\sqrt{3} + 2i$

② $\cos 240^\circ + i\sin 240^\circ \rightarrow r = 1, \ \theta = 240^\circ = \frac{4}{3}\pi$

$x = r\cos\theta = \cos 240 = -\frac{1}{2}$.

$y = r\sin\theta = \cancel{\cos}\sin 240 = -\frac{\sqrt{3}}{2}$.

stan. form $\rightarrow -\frac{1}{2} - \frac{\sqrt{3}}{2}i$

example: Give the polar form of the number:

①.

$3 - 4i \rightarrow x = 3, \ y = -4$.

$~~r = \sqrt{\ldots}~~ \quad r = \sqrt{x^2+y^2}$

$$r = \sqrt{3^2+(-4)^2} = 5$$

$$\theta = \tan^{-1}\left(\frac{-4}{3}\right) = -53.13^\circ$$

polar form $\longrightarrow$ $5(\cos -53.13^\circ + i \sin -53.13)$

$5(\cos 306.87 + i \sin 306.87)$

(2)

$6i \longrightarrow x = 0, \; y = 6$

$$r = \sqrt{x^2+y^2} = \sqrt{0+36} = 6$$

$$\theta = \tan^{-1}\left(\frac{6}{0}\right) = \frac{\pi}{2}$$

Palar form $\longrightarrow$ $6\left(\cos \frac{\pi}{2} + i \sin \frac{\pi}{2}\right)$

(3) $4 \longrightarrow x = 4, \; y = 0$

$$r = \sqrt{x^2+y^2} \longrightarrow r = 4$$

$$\theta = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}\left(\frac{0}{4}\right) = 2\pi \text{ أو } 0$$

polar form $\longrightarrow$ $4(\cos 0 + i \sin 0)$

أو

$4(\cos 2\pi + i \sin 2\pi)$

* العمليات على الأعداد المركبة في الصورة القطبية :

let. $z_1 = r_1 e^{i\theta_1}$, $z_2 = r_2 e^{i\theta_2}$

① عند الضرب :

$$z_1 z_2 = r_1 r_2 e^{i(\theta_1+\theta_2)}$$

$$= r_1 r_2 \left[\cos(\theta_1+\theta_2) + i\sin(\theta_1+\theta_2)\right]$$

② عند القسمة :

$$\frac{z_1}{z_2} = \frac{r_1 e^{i\theta_1}}{r_2 e^{i\theta_2}} = \frac{r_1}{r_2} e^{i(\theta_1-\theta_2)}$$

$$\frac{z_1}{z_2} = \frac{r_1}{r_2}\left[\cos(\theta_1-\theta_2) + i\sin(\theta_1-\theta_2)\right]$$

③ المعكوس الضربي :

$$z_1^{-1} = \frac{1}{r_1}\left[\cos\theta_1 - i\sin\theta_1\right]$$

$$z_2^{-1} = \frac{1}{r_2}\left[\cos\theta_2 - i\sin\theta_2\right]$$

* صيغة ديموافر Demoirre Formula.

اذا كان $z = re^{i\theta}$ وكان n عدد ثابت

فان

$$z^n = (re^{i\theta})^n \longrightarrow r^n e^{in\theta}$$

$$z^n = \left[r(\cos\theta + i\sin\theta)\right]^n = r^n \cos n\theta + i\sin n\theta.$$

example: Find

① $\left[3\left(\cos\frac{5\pi}{6}+i\sin\frac{5\pi}{6}\right)\right]^{4}$

$3^{4}\left[\left(\cos 4\left(\frac{5\pi}{6}\right)+i\sin 4\left(\frac{5\pi}{6}\right)\right)\right]$

$81\ \cos\frac{10\pi}{3}+i\sin\frac{10\pi}{6}$

② $\left[2\left(\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}\right)\right]^{-2}$

$2^{-2}\ \cos\left(-2\times\frac{\pi}{6}\right)+i\sin\left(-2\times\frac{\pi}{6}\right)$

$\frac{1}{4}\left[\cos\frac{\pi}{3}-i\sin\frac{\pi}{3}\right]$

$\frac{1}{4}\left(\frac{1}{2}-\frac{\sqrt{3}}{2}i\right)=\boxed{\frac{1}{8}-\frac{\sqrt{3}}{8}i}$

③ $\left[2\left[\cos\left(\frac{\pi}{6}\right)+i\sin\left(\frac{\pi}{6}\right)\right]\right]^{3}$

$8\ \cos\frac{\pi}{2}+i\sin\frac{\pi}{2}$

$8(0+i)=\boxed{8i}$

example: prove that

$(1+i)^{20}=-1024$

← القطبية

$x=1\ ,\ y=1 \qquad r=\sqrt{2}\ ,\ \theta=45=\frac{\pi}{4}$

polar form $\rightarrow$ $\sqrt{2}\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right)$

$(1+i)^{20}=\left[\sqrt{2}\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right)\right]^{20}$

$$(4+i)^{20} = 2^{10}(\cos 5\pi + i\sin 5\pi)$$

$$= 1024(-1 + (i\times 0))$$

$$(1+i)^{20} = -1024 \;\#$$

example:

① Find $\left[3\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)\right]\left[\frac{1}{3}\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)\right]$

$$\left(3\times\frac{1}{3}\right)\left(\cos\left(\frac{\pi}{3}+\frac{2\pi}{3}\right) + i\sin\left(\frac{\pi}{3}+\frac{2\pi}{3}\right)\right)$$

$$\cos\pi + i\sin\pi = -1 \;\#$$

② $$\frac{12\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)}{3\left(\cos\pi/6 + i\sin\pi/6\right)}$$

$$\frac{12}{3}\left[\cos\left(\frac{\pi}{3}-\frac{\pi}{6}\right) + i\sin\left(\frac{\pi}{3}-\frac{\pi}{6}\right)\right]$$

$$4\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$$

$$4\left(\frac{\sqrt{3}}{2} + i\frac{1}{2}\right) = 2\sqrt{3} - 2i \;\#$$

الجذور النونية للأعداد المركبة

* باستخدام نظرية ديموافير $z^n = r^n(\cos n\theta + i\sin n\theta)$

$$\sqrt[n]{z} = r^{\frac{1}{n}}\left(\cos\frac{\theta+2k\pi}{n} + i\sin\frac{\theta+2k\pi}{n}\right)$$

ex.

$k = 0, 1, 2 \ldots n-1$

اوجد الجذور الرباعية للعدد المركب $-4-4i$

$n = 4$

① نحول العدد المركب إلى الصورة القطبية

$-4-4i$

$x = -4 \quad , y = -4$

$r = \sqrt{(-4)^2 + (-4)^2} = \sqrt{32}$

$\theta = \tan^{-1}\left(\frac{-4}{-4}\right) = \frac{5\pi}{4}$ موجودة في الربع الثالث

$z = r(\cos\theta + i\sin\theta)$

$z = \sqrt{32}\left(\cos\frac{5\pi}{4} + i\sin\frac{5\pi}{4}\right)$

نطبق نظرية ديموفير

$$(\sqrt{32})^{1/4}\left(\cos\left(\frac{\frac{5\pi}{4}+2k\pi}{4}\right) + i\sin\left(\frac{5\pi/4 + 2k\pi}{4}\right)\right)$$

$$32^{1/8}\left(\cos\left(\frac{5\pi}{16}+\frac{k\pi}{2}\right) + i\sin\left(\frac{5\pi}{16}+\frac{k\pi}{2}\right)\right) \quad k = 0, 1, 2, 3$$

at $k = 0$

$$\sqrt[8]{32}\left(\cos\frac{5\pi}{16} + i\sin\frac{5\pi}{16}\right).$$

at $k = 1$

$$\sqrt[8]{32}\left(\cos\left(\frac{5\pi}{16} + \frac{2\pi}{4}\right) + i\sin\left(\frac{5\pi}{16} + \frac{2\pi}{4}\right)\right)$$

$$\sqrt[8]{32}\left(\cos\frac{13\pi}{16}\right) + i\sin\left(\frac{13}{16}\pi\right).$$

at $k = 2$.

$$\sqrt[8]{32}\left(\cos\left(\frac{5\pi}{16} + \pi\right) + i\sin\left(\frac{5\pi}{16} + \pi\right)\right)$$

$$\sqrt[8]{32}\left[\cos\left(\frac{21\pi}{16}\right) + i\sin\left(\frac{21}{16}\pi\right)\right]$$

at $k = 3$.

$$\sqrt[8]{32}\left[\cos\left(\frac{5\pi}{16} + \frac{3\pi}{2}\right) + i\sin\left(\frac{5\pi}{16} + \frac{3\pi}{2}\right)\right]$$

$$\sqrt[8]{32}\left[\cos\left(\frac{29}{16}\pi\right) + i\sin\left(\frac{29\pi}{16}\right)\right].$$

Exercisec

Q1: Find stan. form $x + iy$.

① $(3+4i)(1+2i)$

$3 + 6i + 4i - 8$

$-5 + 10i \rightarrow$ stan. form.

② $\dfrac{4}{1-i} + \dfrac{2-i}{1+i} + 5i$

$\dfrac{4}{1-i} \times \dfrac{1+i}{1+i} + \dfrac{(2-i)}{1+i} \times \dfrac{(1-i)}{1-i} + 5i$

$\dfrac{4+4i}{2} + \dfrac{2-2i-i+1}{2} + 5i$

$\dfrac{4+4i+2-3i+1}{2} + 5i$

$\dfrac{7+i}{2} + \dfrac{5i}{1} = \dfrac{7+i+10i}{2}$

$\dfrac{7+11i}{2} = \dfrac{7}{2} + \dfrac{11i}{2} \rightarrow$ stan. form

③ $(2i-1)^2 + 2\left(\dfrac{1-i}{5i}\right)^3$

* نحول $\dfrac{1-i}{5i}$ الى الصورة المناسبة ؟

$\dfrac{1-i}{5i} \cdot \dfrac{-5i}{-5i} = \dfrac{-5i-5}{25} = \dfrac{-1}{5} - \dfrac{1}{5}i$

$x = \dfrac{-1}{5}$, $y = \dfrac{1}{5}$

$r = \sqrt{x^2+y^2} \longrightarrow \sqrt{(-1/5)^2+(1/5)^2} \longrightarrow \boxed{r = \frac{\sqrt{2}}{5}}$

$\frac{5\pi}{4}$

$\theta = \tan^{-1}\left(\frac{-1/5}{-1/5}\right) \rightarrow \tan^{-1}(1) = 45 \rightarrow$ الربع الثالث $180+45 = 225°$

$r(\cos\theta + i\sin\theta) \rightarrow \frac{\sqrt{2}}{5}\left(\cos\frac{5\pi}{4} + i\sin\frac{5\pi}{4}\right)$

$\left(\frac{1-i}{5i}\right)^3 = \left(\frac{\sqrt{2}}{5}\right)^3\left(\cos(3)\frac{5\pi}{4} + i\sin(3)\frac{5\pi}{4}\right)$

$= \frac{2\sqrt{2}}{125}\left(\cos\frac{15}{4}\pi + i\sin\frac{15}{4}\pi\right)$

$x = r\cos\theta = \frac{2\sqrt{2}}{125}\cos\frac{15\pi}{4} = \frac{2}{125}.$

$y = r\sin\theta = \frac{2\sqrt{2}}{125}\sin\frac{15}{4}\pi = \frac{2}{125}.$

$\left(\frac{1-i}{5i}\right)^3 = \boxed{\frac{2}{125} + \frac{2}{125}i}$

$(2i-1)^2 + 2\left(\frac{2}{125} + \frac{2}{125}i\right).$

$-4 - 4i + 1 + \frac{4}{125} + \frac{4}{125}i$

$\boxed{\frac{-371}{125} \quad \frac{504}{125}i}$

Q₂: Find

① $\left|\frac{4}{1-2i} + \frac{2-i}{1-i}\right|$

$\frac{4}{1-2i} + \frac{2-i}{1-i} = \frac{4}{1-2i}\cdot\frac{1+2i}{1+2i} + \frac{(2-i)}{1-i}\frac{(1+i)}{1+i}$

$= \frac{4+8i}{5} + \frac{2+2i-i+1}{2}$

$$= \frac{4+8i}{5} + \frac{3+i}{2} = \frac{8+16i+15+5i}{10} = \frac{23+21i}{10}.$$

$$\frac{4}{1-2i} + \frac{2-i}{1-i} = \underbrace{\frac{23}{10}}_{x} + \underbrace{\frac{21}{10}}_{y} i \leftarrow \text{stand. form.}$$

$$|Z| = \sqrt{x^2+y^2}$$

$$\left|\frac{4}{1-2i} + \frac{2-i}{1-i}\right| = \sqrt{\left(\frac{23}{10}\right)^2 + \left(\frac{21}{10}\right)^2} = \boxed{\sqrt{\frac{97}{10}}}$$ #

② $Re\left\{\frac{(1+2i)^2}{3-4i}\right\}$

$$\frac{(1+2i)^2}{3-4i} = \frac{1+4i-4}{3-4i} = \frac{-3+4i}{3-4i} \cdot \frac{3+4i}{3+4i}$$

$$\frac{(-3+4i)(3+4i)}{(3-4i)(3+4i)} = \boxed{-1}$$ #

③ $\arg(3-3i)$

$x = 3, \quad y = -3$

$\theta = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}\left(\frac{-3}{3}\right) = -45°$ تقع في الربع الرابع

$360 - 45 = 315°$.

$\arg(3-3i) = \frac{7\pi}{4}$

Q_3: Convert ~~the~~ to polar form.

$$2i + \left(\frac{1+i}{1-i}\right)^3$$

$$\frac{1+i}{1-i} \cdot \frac{1+i}{1+i} = \frac{1+2i-1}{2} = i$$

$$2i + i^3 = 2i - i = \boxed{i} \longrightarrow \text{stand. form}$$

$$x = 0 \ , \ y = 1$$

$$r = \sqrt{x^2+y^2} = r = \sqrt{1} = \boxed{r=1}$$

$$\theta = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}\left(\frac{1}{0}\right) = 90 = \boxed{\frac{\pi}{2}}$$

$$r(\cos\theta + i\sin\theta) = \cos\frac{\pi}{2} + i\sin\frac{\pi}{2} \ \#$$

Q_4: solve the equation to Find x, y (Real Values).

① $(3+4i)^2 - 2(x - iy) = x + iy$

$$9 + 24i - 16 - 2x + 2iy = x + iy$$

$$-7 + 24i + 2iy = 3x + iy$$

$$-7 = 3x \longrightarrow \boxed{x = -7/3}$$

$$24i + 2iy = iy \longrightarrow \boxed{y = -24}$$

(2) $(3-2i)(x+iy) = 2(x-2iy) + 2i - 1$:

$3x + 3iy - 2ix + 2y = 2x - 4iy + 2i - 1$:

$3x + 2y = 2x - 1 \rightarrow \boxed{x = -1-2y} \rightarrow (1)$

$3iy - 2ix = -4iy + 2i \rightarrow 3y - 2x = -4y + 2.$

$7y = 2 + 2x \rightarrow \boxed{x+1 = \frac{7}{2}y} \rightarrow (2)$

(1) to (3) $x = -1-2y \rightarrow \boxed{x+1 = -2y} \rightarrow (3)$

$\frac{7}{2}y = -2y \rightarrow \boxed{y=0}$

sub. y in (3) $x+1 = -2(0) \rightarrow x+1 = 0$

$\boxed{x = -1}$

(3) $\left(\frac{1+i}{1-i}\right)^2 + \frac{1}{x+iy} = 1+i$, Ans. $x = \frac{2}{5}$, $y = \frac{-1}{5}$

Q_5: $a = \frac{7-i}{2-i}$, $b = \frac{13-i}{4+i}$

(1) prove that a, b Two conjugate numbers:
(مترافقان)

(2) prove $\frac{a^3+b^3}{a^2+b^2} = \frac{9}{4}$

(1) $a = \frac{7-i}{2-i} \cdot \frac{2+i}{2+i}$

$a = \frac{14+7i-2i+1}{4+1} \longrightarrow a = \frac{15+5i}{5}.$

$a = \frac{15}{5} + \frac{5}{5} i \longrightarrow \boxed{a = 3 + i}$

$b = \frac{13 - i}{4 + i} \cdot \frac{4 - i}{4 - i} \longrightarrow b = \frac{52 - 13i - 4i - 1}{16 + 1}$

$b = \frac{51 - 17i}{17} \longrightarrow b = \frac{51}{17} - \frac{17}{17} i$

$\boxed{b = 3 - i}$

$\therefore a, b$ conjugate.

(2) $\frac{a^3 + b^3}{a^2 + b^2} = \frac{9}{4}$

$a = 3 + i$
$b = 3 - i$

$a + b = 6$
$a \cdot b = (3 + i)(3 - i) = 10$

$a^3 + b^3 = (a+b)^3 - 3ab(a+b)$
$a^2 + b^2 = (a+b)^2 - 2ab$

تذكر

$a^3 + b^3 = (a+b)^3 - 3ab(a+b)$

$= 6^3 - (3 \times 10 \times 6)$

$a^3 + b^3 = 216 - 180 = \underline{36}$

$a^2 + b^2 = (a+b)^2 - 2ab$

$= 6^2 - (2 \times 10)$

$a^2 + b^2 = 36 - 20 = \underline{16}$

$\frac{a^3 + b^3}{a^2 + b^2} = \frac{36}{16} = \frac{9}{4}$ #

Q6: if $a = 2 + i$, $b = 3 - 4i$ prove that

$$\frac{(\bar{a})^2}{b} = 1$$

$a = 2 + i$, $\bar{a} = 2 - i$

$$\frac{(2-i)^2}{3+4i} = \frac{4 - 4i - 1}{3+4i} \longrightarrow \frac{3+4i}{3+4i} = 1$$

Q7: if $2 - 4i$ one the roots of the equation
$2x^2 - x - bx + c - 6 = 0$. Find the value $a, b \in \mathbb{R}$.

root root

$Z_1 = 2 - 4i$, $Z_2 = 2 + 4i$ الجذر الآخر

$x^2 + bx + c = 0$. ← الصورة العامة للمعادلة التربيعية

$2x^2 - (1+b)x + (c-6) = 0$. $\div 2$

$$x^2 - \left(\frac{1+b}{2}\right)x + \left(\frac{c-6}{2}\right) = 0.$$

$$Z_1 + Z_2 = b \longrightarrow (2-4i)(2+4i) = \frac{1+b}{2}$$

$$4 = \frac{1+b}{2} \longrightarrow b = 7$$

$$Z_1 \cdot Z_2 = c \longrightarrow (2-4i)(2+4i) = \frac{c-6}{2}$$

$$4 + 16 = \frac{c-6}{2} \longrightarrow c = 46$$