كنتك الهندسة
ENGINEERING

كلية الهندسة — جامعة مصراتة

القسم العام

رياضة "2"

الشيت الرابع

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10 * التعويضات المثلثية :

نستخدم هذه الطريقة في حالة احتواء التكامل على احدى الصور:

$$a^2 - u^2 \quad \text{or} \quad a^2 + u^2 \quad \text{or} \quad u^2 - a^2.$$

if $a^2 - u^2 \rightarrow$ نفرض $u = a\sin\theta$ or $u = a\tanh\theta$

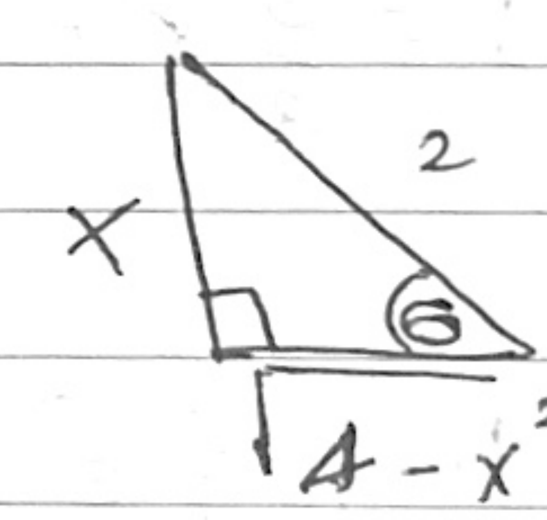
if $a^2 + u^2 \rightarrow$ نفرض $u = a\tan\theta$ or $u = a\sinh\theta$

if $u^2 - a^2 \rightarrow$ نفرض $u = a\sec\theta$ or $u = a\cosh\theta$

Q_{10}: Find the following int.

① $\displaystyle\int \frac{x+1}{\sqrt{4-x^2}}\,dx.$

نفرض $x = 2\sin\theta \rightarrow dx = 2\cos\theta\, d\theta.$



$$\int \frac{2\sin\theta + 1}{\sqrt{4 - 4\sin^2\theta}}\, 2\cos\theta\, d\theta.$$

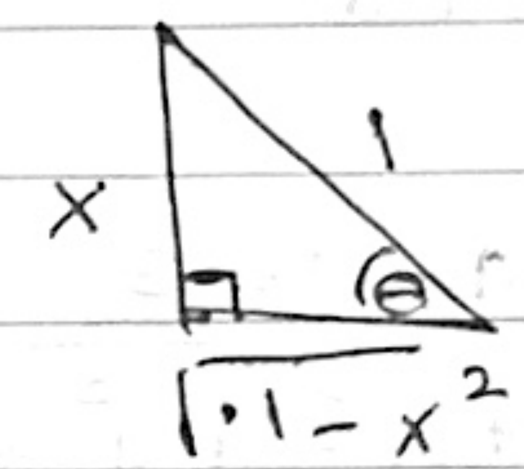
$$\int \frac{2\sin\theta + 1}{2\sqrt{\cos^2\theta}}\, 2\cos\theta\, d\theta. \Rightarrow \int \frac{2\sin\theta + 1}{2\cos\theta}\, 2\cos\theta\, d\theta$$

$$\int (2\sin\theta + 1)\, d\theta = -2\cos\theta + \theta + C.$$

$$I = -2 \cdot \frac{\sqrt{4-x^2}}{2} + \sin^{-1}\frac{x}{2} + C. = -\sqrt{4-x^2} + \sin^{-1}\frac{x}{2} + C$$

② $\int (1-x^2)^{3/2}\, dx$

نفرض $x = \sin\theta \rightarrow dx = \cos\theta \, d\theta$



$$\int (1-\sin^2\theta)^{3/2} \cdot \cos\theta \, d\theta$$

$$= \int (\cos^2\theta)^{3/2} \cdot \cos\theta \, d\theta = \int \cos^4\theta \, d\theta$$

$$= \int \left(\frac{1+\cos 2\theta}{2}\right)^2 d\theta = \frac{1}{4}\int (1 + 2\cos 2\theta + \cos^2 2\theta) \, d\theta$$

$$= \frac{1}{4}\int \left[1 + 2\cos 2\theta + \frac{1}{2}(1+\cos 4\theta)\right] d\theta$$

$$= \frac{1}{4}\int \left(\frac{3}{2} + 2\cos 2\theta + \frac{1}{2}\cos 4\theta\right) d\theta$$

$$= \frac{1}{4}\left(\frac{3}{2}\theta + \sin 2\theta + \frac{1}{8}\sin 4\theta\right) + c$$

$$= \frac{1}{4}\left(\frac{3}{2}\theta + 2\sin \cancel{2}\theta \cos \cancel{2}\theta + \frac{1}{8} \cdot 2\sin 2\theta \cos 2\theta\right) + c$$

$$= \frac{1}{4}\left(\frac{3}{2}\theta + 2\sin\theta\cos\theta + \frac{1}{4} \cdot 2\sin\theta\cos\theta \cdot (\cos^2\theta - \sin^2\theta)\right) + c$$

$$= \frac{1}{4}\left(\frac{3}{2}\sin^{-1}x + 2x\sqrt{1-x^2} + \frac{1}{2}x\sqrt{1-x^2}\,(1-x^2-x^2)\right) + c$$

$$= \frac{1}{4}\left(\frac{3}{2}\sin^{-1}x + 2x\sqrt{1-x^2} + \frac{1}{2}x\sqrt{1-x^2}\,(1-2x^2)\right) + c$$

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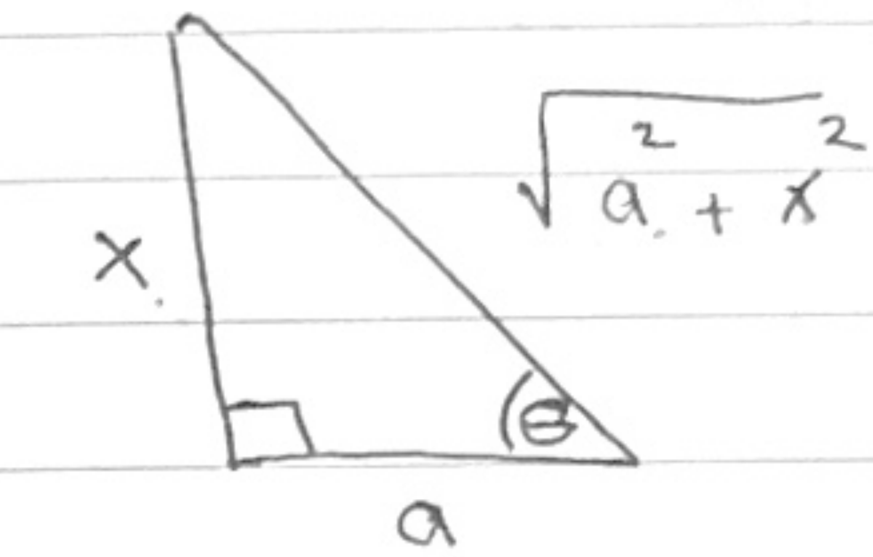
③ $\displaystyle\int \frac{dx}{x\sqrt{a^2+x^2}}$ نفرض $x = a\tan\theta \rightarrow dx = a\sec^2\theta \, d\theta.$

$$I = \int \frac{a\sec^2\theta}{a\tan\theta\sqrt{a^2+a^2\tan^2\theta}} \cdot d\theta$$



$$= \frac{1}{a}\int \frac{\sec^2\theta}{\tan\theta \cdot \sec\theta} \cdot d\theta$$

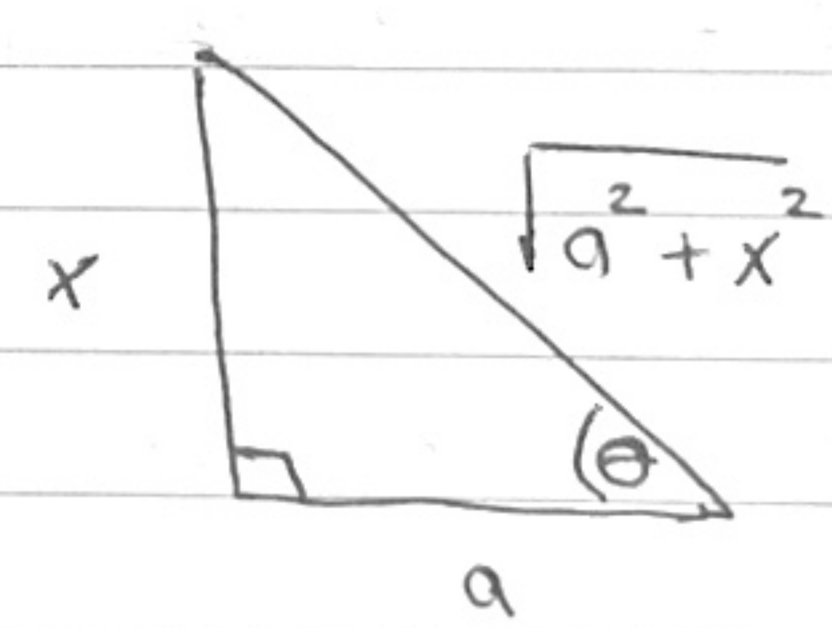
$$= \frac{1}{a}\int \frac{\sec\theta}{\tan\theta} d\theta = \frac{1}{a}\int \frac{1}{\cos\theta} \cdot \frac{\cos\theta}{\sin\theta} d\theta.$$

$$= \frac{1}{a}\int \csc\theta \, d\theta = \frac{1}{a}\ln\left|\csc\theta - \cot\theta\right| + c.$$

$$I = \frac{1}{a}\ln\left|\frac{\sqrt{a^2+x^2}}{x} - \frac{a}{x}\right| + c$$

④ $\displaystyle\int \frac{dx}{(a^2+x^2)^2}$, نفرض $x = a\tan\theta \rightarrow dx = a\sec^2\theta \, d\theta.$

$$I = \int \frac{a\sec^2\theta \, d\theta}{(a^2+a^2\tan^2\theta)^2}$$



$$= \int \frac{a\sec^2\theta}{a^4\sec^4\theta} d\theta. = \frac{1}{a^3}\int \frac{1}{\sec^2\theta} d\theta.$$

$$\frac{1}{a^3}\int \cos^2\theta \, d\theta = \frac{1}{a^3}\int \frac{1}{2}(1+\cos 2\theta) \, d\theta.$$

$$= \frac{1}{2a^3}\left[\theta + \frac{1}{2}\sin 2\theta\right] + c = \frac{1}{2a^3}\left[\theta + \sin\theta\cos\theta\right] + c.$$

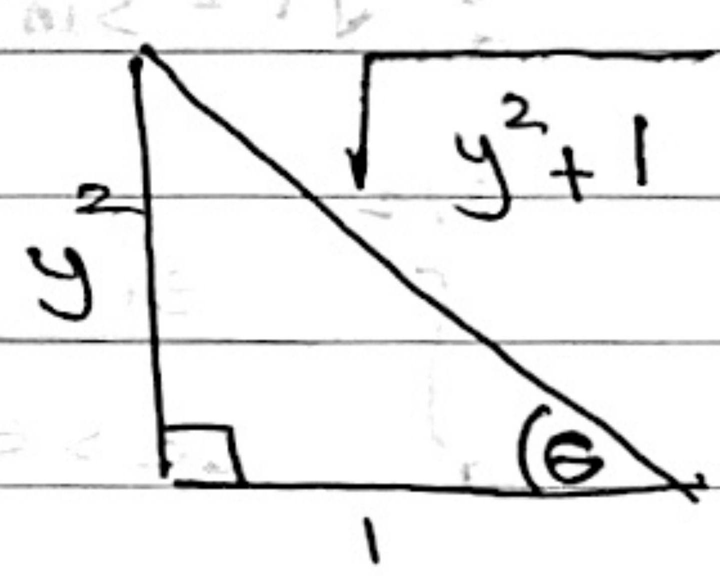
$$I = \frac{1}{2a^3}\left[\tan^{-1}\frac{x}{a} + \frac{x}{\sqrt{a^2+x^2}}\cdot\frac{a}{\sqrt{a^2+x^2}}\right] + c.$$

$$I = \frac{1}{2a^3}\left[\tan^{-1}\frac{x}{a} + \frac{ax}{x^2+a^2}\right] + c.$$

(5) $\int \frac{dy}{y^4+1}$, نفرض $y^2 = \tan\theta$, $2y\,dy = \sec^2\theta\,d\theta.$

$$y\,dy = \frac{1}{2}\sec^2\theta\,d\theta.$$

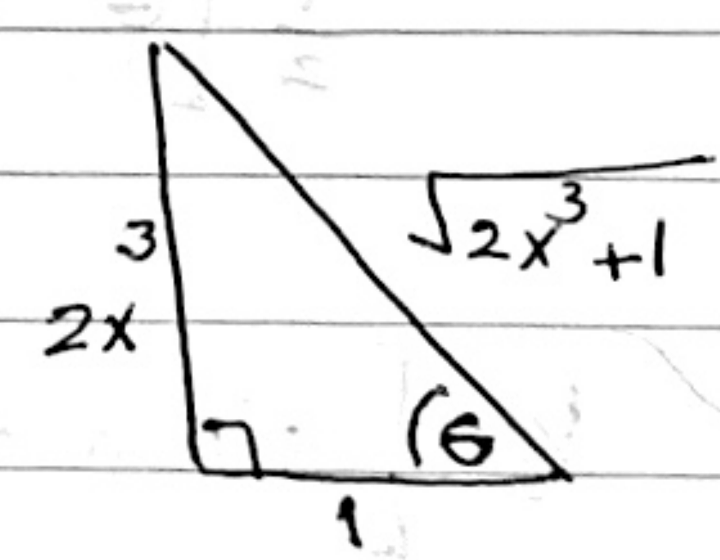
$$\int \frac{\frac{1}{2}\sec^2\theta\,d\theta}{\tan^2\theta+1} = \frac{1}{2}\int\frac{\sec^2\theta}{\sec^2\theta}\,d\theta.$$



$$\frac{1}{2}\int d\theta = \frac{1}{2}\theta + c. = \frac{1}{2}\tan^{-1}y^2 + c.$$

(6) $\int \frac{x^2}{1+4x^6}\,dx$, نفرض $2x^3 = \tan\theta. \rightarrow 6x^2\,dx = \sec^2\theta\,d\theta$

$$\int \frac{\frac{1}{6}\sec^2\theta}{1+\tan^2\theta}\,d\theta = \frac{1}{6}\int\frac{\sec^2\theta}{\sec^2\theta}\,d\theta = \frac{1}{6}\int d\theta$$



$$I = \frac{1}{6}\theta + c.$$

$$I = \frac{1}{6}\tan^{-1}(2x^3 + c$$

(7) $\int \frac{dx}{x\sqrt{x^2-a^2}}$, نفرض $x = a\sec\theta \rightarrow dx = a\sec\theta \cdot \tan\theta \, d\theta$

$$\int \frac{a\sec\theta\tan\theta}{a\sec\theta\sqrt{a^2\sec^2\theta - a^2}}\,d\theta \rightarrow \int \frac{\tan\theta}{a\tan\theta}\,d\theta$$

$$\int \frac{1}{a}\,d\theta = \frac{1}{a}\theta + C = \frac{1}{a}\sec^{-1}\left(\frac{x}{a}\right) + C.$$

(8) $\int_0^{1/2} \frac{x^2}{\sqrt{1-x^2}}\,dx$, نفرض $x = \sin\theta \rightarrow dx = \cos\theta\,d\theta$

$$\int_0^{1/2} \frac{\sin^2\theta}{\sqrt{1-\sin^2\theta}}\cos\theta\,d\theta$$

$$\int_0^{1/2} \frac{\sin^2\theta}{\cos\theta}\cdot\cos\theta\,d\theta \rightarrow \int_0^{1/2} \sin^2\theta\,d\theta$$

$$\int \frac{1}{2}(1-\cos 2\theta)\,d\theta = \frac{1}{2}\left[\theta - \frac{1}{2}\sin 2\theta\right]_0^{1/2}$$

$$\frac{1}{2}\left[\theta - \sin\theta\cos\theta\right]_0^{1/2}$$

$$\frac{1}{2}\left[\sin^{-1}x - x\sqrt{1-x^2}\right]_0^{1/2} = \frac{1}{2}\left[\sin^{-1}\frac{1}{2} - \frac{1}{2}\sqrt{1-\frac{1}{4}}\right]$$

$$\boxed{I = \frac{1}{2}\left(\frac{\pi}{6} - \frac{\sqrt{3}}{4}\right)}$$

* تفاضل الدوال المثلثية العكسية :

الدالة	$\longrightarrow$ تفاضلها
$\sin^{-1} u$	$\dfrac{1}{\sqrt{1-u^2}}\, u'$
$\cos^{-1} u$	$\dfrac{-1}{\sqrt{1-u^2}} \cdot u'$
$\tan^{-1} u$	$\dfrac{1}{1+u^2}\, u'$
$\cot^{-1} u$	$\dfrac{-1}{1+u^2} \cdot u'$
$\sec^{-1} u$	$\dfrac{1}{u\sqrt{u^2-1}} \cdot u'$
$\csc^{-1} u$	$\dfrac{-1}{u\sqrt{u^2-1}}\, u'$

Q11: Find the following

① $y = \sin^{-1}\sqrt{x} \rightarrow y' = \dfrac{1}{\sqrt{1-x}} \cdot \dfrac{1}{2\sqrt{x}} \rightarrow \dfrac{1}{2\sqrt{x}\sqrt{1-x}}$.

② $y = \tan^{-1}(\cos x) \rightarrow y' = \dfrac{1}{1+\cos^2 x} \cdot -\sin x$.

③ $y = \ln|\cot^{-1} 3x| \rightarrow y' = \dfrac{1}{\cot^{-1} 3x} \cdot \dfrac{-1}{1+(3x)^2} \cdot 3$

④ $y = \tan^{-1} e^{\sin x} \rightarrow y' = \dfrac{1}{1+(e^{\sin x})^2} \cdot e^{\sin x} \cdot \cos x$.

⑤ $y = \cot^{-1} \dfrac{x}{1-x} \rightarrow y' = \dfrac{-1}{1+\left(\dfrac{x}{1-x}\right)^2} \cdot \dfrac{(1-x)\cdot(1) - x(-1)}{(1-x)^2}$

* تفاضل الدوال الزائدية .

$\sinh^{-1} u \qquad \dfrac{1}{\sqrt{u^2+1}}\,u'$

$\cosh^{-1} u \qquad \dfrac{1}{\sqrt{u^2-1}}\,u'$

$\tanh^{-1} u \qquad \dfrac{1}{1-u^2}\,u'$

$\coth^{-1} u \qquad \dfrac{1}{1-u^2}\cdot u'$

$\operatorname{sech}^{-1} u \qquad \dfrac{-1}{u\sqrt{1-u^2}}\cdot u'$

$\operatorname{csch}^{-1} u \qquad \dfrac{-1}{u\sqrt{1+u^2}}\,u'$

Q_{12}: Find the folloing Diff.

① $y = \cosh^{-1} e^{x}. \longrightarrow y' = \dfrac{1}{\sqrt{e^{2x}-1}}\cdot e^{x}.$

② $y = \sinh^{-1} x + \cosh^{-1} x. \longrightarrow y' = \dfrac{1}{\sqrt{x^2+1}}\cdot 1 + \dfrac{1}{\sqrt{x^2-1}}\cdot 1$

③ $y = \sqrt[3]{\sinh^{-1} x}. \longrightarrow y' = \frac{1}{3}(\sinh^{-1} x)^{-2/3}\cdot\dfrac{1}{\sqrt{x^2+1}}\cdot 1$

④ $y = \tan^{-1}(\sin(\ln x)) \longrightarrow y' = \dfrac{1}{1-(\sin(\ln x))^2}\cdot\cos(\ln x\cdot\dfrac{1}{x} \rightarrow y' = \dfrac{\sec\ln x}{x}.$

⑤ $y = e^{\operatorname{sech}^{-1} x} \longrightarrow y' = e^{\operatorname{sech}^{-1} x}\cdot\dfrac{-1}{x\sqrt{1-x^2}}\cdot 1 \rightarrow y' = \dfrac{-e^{\operatorname{sech}^{-1} x}}{x\sqrt{1-x^2}}$

* تكاملات ناتجها دوال عكسية :

Q13: Find the following int.

① $\int \frac{dx}{x^2+16} \rightarrow \frac{1}{16}\int \frac{1}{\frac{x^2}{16}+1}\,dx$

$\frac{1}{16}\int \frac{1}{(\frac{x}{4})^2+1}\,dx \quad = \frac{1}{4}\int \frac{\frac{1}{4}}{(\frac{x}{4})^2+1}\,dx.$

$= \frac{1}{4}\tan^{-1}(\frac{x}{4}) + c.$

② $\int \frac{dx}{x^2+9} \longrightarrow \frac{1}{9}\int \frac{1}{\frac{x^2}{9}+1}\,dx.$

$= \frac{1}{3}\int \frac{\frac{1}{3}}{(\frac{x^2}{3})+1}\,dx$

$= \frac{1}{3}\tan^{-1}(\frac{x}{2}) + c.$

③ $\int \frac{dx}{2x^2+9} = \frac{1}{9}\int \frac{1}{\frac{2x^2}{9}+1}\,dx.$

$\frac{1}{9}\int \frac{1}{(\frac{\sqrt{2}x}{3})^2+1}\,dx. \quad = \frac{1}{3\sqrt{2}}\int \frac{\frac{\sqrt{2}}{3}}{(\frac{\sqrt{2}x}{3})^2+1}\,dx.$

$= \frac{1}{3\sqrt{2}}\tan^{-1}(\frac{\sqrt{2}x}{3}) + c.$

④ $\int \frac{dx}{\sqrt{4-x^2}} = \frac{1}{2}\int \frac{dx}{\sqrt{1-\frac{x^2}{4}}}$

$= \frac{1}{2}\int \frac{1}{\sqrt{1-\left(\frac{x}{2}\right)^2}}\, dx = \int \frac{\frac{1}{2}}{\sqrt{1-\left(\frac{x}{2}\right)^2}}\, dx.$

$= \sin^{-1}\left(\frac{x}{2}\right) + c.$

⑤ $\int \frac{dx}{\sqrt{3-2x^2}} = \frac{1}{\sqrt{3}}\int \frac{dx}{\sqrt{1-\frac{2x^2}{3}}}$

$\frac{1}{\sqrt{3}}\int \frac{1}{1-\left(\frac{\sqrt{2}x}{\sqrt{3}}\right)^2}\, dx, = \frac{1}{\sqrt{2}}\int \frac{\frac{\sqrt{2}}{\sqrt{3}}}{1-\left(\frac{\sqrt{2}x}{\sqrt{3}}\right)^2}\, dx$

$= \frac{1}{\sqrt{2}} \sin^{-1}\left(\frac{\sqrt{2}x}{\sqrt{3}}\right) + c.$

⑥ $\int \frac{-dx}{\sqrt{5-x^2}} = \frac{1}{\sqrt{5}}\int \frac{-dx}{\sqrt{1-\frac{x^2}{5}}}$

$\frac{1}{\sqrt{5}}\int \frac{-dx}{\sqrt{1-\left(\frac{x}{\sqrt{5}}\right)^2}} = \int \frac{-1/\sqrt{5}}{\sqrt{1-\left(\frac{x}{\sqrt{5}}\right)^2}}\, dx,$

$= \cos^{-1}\left(\frac{x}{\sqrt{5}}\right) + c.$

⑦ $\int \frac{dx}{x\sqrt{2x^2-3}} = \frac{1}{\sqrt{3}} \int \frac{dx}{x\sqrt{\frac{2x^2}{3}-1}}$

$\frac{1}{\sqrt{3}} \int \frac{\frac{\sqrt{2}}{\sqrt{3}}\,dx}{\frac{\sqrt{2}}{\sqrt{x}}\,\cancel{x}\sqrt{\left(\frac{\sqrt{2}x}{\sqrt{3}}\right)^2-1}}$

$\frac{1}{\sqrt{3}} \sec^{-1}\left(\frac{\sqrt{2}x}{\sqrt{3}}\right) + c.$

⑧ $\int \frac{x^2}{\sqrt{25-x^6}}\,dx = \frac{1}{5} \int \frac{x^2}{\sqrt{1-\left(\frac{x^3}{5}\right)^2}}\,dx.$

$\frac{1}{3} \int \frac{\frac{3}{5}x^2}{\sqrt{1-\left(\frac{x^3}{5}\right)^2}}\,dx = \frac{1}{3}\sin^{-1}\left(\frac{x^3}{5}\right) + c.$

⑨ $\int \frac{\sin x}{\cos^2 x + 4}\,dx = \frac{1}{4} \int \frac{\sin x}{\frac{\cos^2 x}{4} + 1}\,dx$

$\frac{1}{4} \int \frac{\sin x}{\left(\frac{\cos x}{2}\right)^2 + 1}\,dx = -\frac{1}{2} \int \frac{-\frac{1}{2}\sin x}{\left(\frac{\cos x}{2}\right)^2+1}\,dx.$

$= \frac{-1}{2}\tan^{-1}\left(\frac{\cos x}{2}\right) + c$

(10) $$\int \frac{\sec x \tan x}{\sec^2 x + 1}\, dx = \int \frac{\sec x \tan x}{(\sec x)^2 + 1}\, dx.$$

$$I = \tan^{-1}(\sec x) + c.$$

(11) $$\int \frac{\csc x \cot x}{\csc^2 x + 1}\, dx = -\int \frac{-\csc x \cot x}{(\csc x)^2 + 1}\, dx.$$

$$I = -\tan^{-1}(\csc x) + c.$$

(12) $$\int \frac{-\sec^2 x}{\sqrt{4 - \tan^2 x}}\, dx = \frac{1}{2}\int \frac{-\sec^2 x}{\sqrt{1 - \frac{\tan^2 x}{4}}}\, dx$$

$$= \frac{1}{2}\int \frac{-\sec^2 x}{\sqrt{1 - \left(\frac{\tan x}{2}\right)^2}}\, dx = \int \frac{-\frac{1}{2}\sec^2 x}{\sqrt{1 - \left(\frac{\tan x}{2}\right)^2}}\, dx.$$

$$I = \cos^{-1}\left(\frac{\tan x}{2}\right) + c.$$

(13) $$\int \frac{dx}{\cos^2 x \sqrt{3 - \tan^2 x}} = \frac{1}{\sqrt{3}}\int \frac{\sec^2 x}{\sqrt{1 - \frac{\tan^2 x}{3}}}\, dx.$$

$$\int \frac{\frac{1}{\sqrt{3}}\sec^2 x}{\sqrt{1 - \left(\frac{\tan x}{\sqrt{3}}\right)^2}}\, dx = \sin^{-1}\left(\frac{\tan x}{\sqrt{x}}\right) + c.$$

(14) $\int \frac{e^x}{e^{2x}+1}\,dx = \int \frac{e^x}{(e^x)^2+1}\,dx.$

$= \tan^{-1} e^x + c.$

(15) $\int \frac{e^{-2x}}{e^{-4x}+2}\,dx. = \frac{1}{2}\int \frac{e^{-2x}}{\left(\frac{e^{-2x}}{\sqrt{2}}\right)^2+1}\,dx.$

$\frac{1}{2} \times \frac{\sqrt{2}}{-2} \int \frac{\frac{-2}{\sqrt{2}}\, e^{-2x}}{\left(\frac{e^{-2x}}{\sqrt{2}}\right)^2+1}\,dx.$

$-\frac{\sqrt{2}}{4} \tan^{-1}\left(\frac{e^{-2x}}{\sqrt{2}}\right) + c.$

(16) $\int \frac{-2^x}{2^{2x}+1}\,dx = \int \frac{-2^x}{(2^x)^2+1}\,dx.$

$= \frac{1}{\ln 2} \int \frac{-2^x \ln 2}{(2^x)^2+1}\,dx. = \frac{1}{\ln 2} \cot^{-1}(2^x) + c$

(17) $\int \frac{3^x}{9^x+2}\,dx = \frac{1}{2}\int \frac{3^x}{\frac{3^{2x}}{2}+1}\,dx.$

$= \frac{1}{2}\int \frac{3^x}{\left(\frac{3^x}{\sqrt{2}}\right)^2+1}\,dx = \frac{1}{\sqrt{2}\ln 3} \int \frac{\frac{1}{\sqrt{2}}\, 3^x \ln 3}{3}$

$= \frac{1}{\sqrt{2}\ln 3} \tan^{-1}\left(\frac{3^x}{\sqrt{2}}\right) + C$

(18) $\int \frac{dx}{\sqrt{e^{2x}+1}} \cdot \frac{e^x}{e^x} = \int \frac{e^x}{e^x\sqrt{(e^x)^2-1}}\, dx.$

$= \sec^{-1}(e^x) + c.$

(19) $\int \frac{-dx}{\sqrt{3^{2x}-1}} \times \frac{3^x}{3^x} = \frac{1}{\ln 3} \int \frac{-3^x \cdot \ln 3}{3^x\sqrt{(3^x)^2-1}}\, dx.$

$= \frac{1}{\ln 3} \csc^{-1}(3^x) + c.$

(20) $\int \frac{dx}{x(1+(\ln x)^2)} = \int \frac{\frac{1}{x}}{1+(\ln x)^2}\, dx.$

$= \tan^{-1}(\ln x) + c.$

(21) $\int \frac{\cos x}{\sqrt{4+\cos^2 x}}\, dx = \int \frac{\cos x}{\sqrt{4+1-\sin^2 x}}\, dx$

$\int \frac{\cos x}{\sqrt{5-\sin^2 x}}\, dx = \frac{1}{\sqrt{5}} \int \frac{\cos x}{\sqrt{1-\frac{\sin^2 x}{5}}}\, dx$

$\frac{1}{\sqrt{5}} \int \frac{\cos x}{\sqrt{1-\left(\frac{\sin x}{\sqrt{5}}\right)^2}}\, dx. = \int \frac{\frac{1}{\sqrt{5}}\cos x}{\sqrt{1-\left(\frac{\sin x}{\sqrt{5}}\right)^2}}.$

$= \sin^{-1}\left(\frac{\sin x}{\sqrt{5}}\right) + c.$

(22) $\displaystyle\int \frac{\sin 2x}{1+\sin^4 x}\, dx = \int \frac{2\sin x \cos x}{1+(\sin^2 x)^2}\, dx.$

$= \tan^{-1}(\sin^2 x) + c.$

(23) $\displaystyle\int \frac{1+x}{\sqrt{1-x^2}}\, dx = \int \frac{1}{\sqrt{1-x^2}}\, dx + \int \frac{x}{\sqrt{1-x^2}}\, dx.$

$\displaystyle\int \frac{1}{\sqrt{1-x^2}}\, dx - \frac{1}{2}\int -2x\,(1-x^2)^{-1/2}\, dx.$

$\sin^{-1} x - (1-x^2)^{1/2} + c = \sin^{-1} x - \sqrt{1-x^2} + c.$

(24) $\displaystyle\int \frac{1}{\sqrt{x^3-x^2}}\, dx = \int \frac{1}{x\sqrt{x-1}}\, dx.$

$\displaystyle= \int \frac{1/\sqrt{x}}{\sqrt{x}\sqrt{(\sqrt{x})^2-1}}\, dx. = 2\int \frac{\frac{1}{2\sqrt{x}}}{\sqrt{x}\sqrt{(\sqrt{x})^2-1}}\, dx.$

$= 2\sec^{-1}(\sqrt{x}) + c.$

(25) $\displaystyle\int \frac{x - \sec^{-1}(x)}{x\sqrt{x^2-1}}\, dx.$

$\displaystyle\int \frac{1}{\sqrt{x^2-1}}\, dx - \int \frac{\sec^{-1}(x)}{x\sqrt{x^2-1}}\, dx.$

$= \cosh^{-1}(x) - \frac{1}{2}(\sec^{-1} x)^2 + c.$

(26) $\int \frac{dx}{x - x\ln^2 x} = \int \frac{1}{x(1-\ln^2 x)} dx$

$= \int \frac{1/x}{1-\ln^2 x} dx. = \tanh^{-1}(\ln x) + c.$

(27) $\int \frac{dx}{x\sqrt{5+2x^4}} = \int \frac{x}{x^2\sqrt{5+2x^4}} dx.$

$\frac{1}{\sqrt{5}} \int \frac{x}{x^2\sqrt{1+\frac{2x^4}{5}}} dx. \rightarrow \frac{1}{\sqrt{5}} \int \frac{x}{x^2\sqrt{1+\left(\frac{\sqrt{2}x^2}{\sqrt{5}}\right)^2}} dx.$

$\frac{1}{2\sqrt{5}} \int \frac{2\frac{\sqrt{2}}{\sqrt{5}}x}{\frac{\sqrt{2}}{\sqrt{5}}x^2\sqrt{1+\left(\frac{\sqrt{2}}{\sqrt{5}}x\right)^2}} dx.$

$\frac{1}{2\sqrt{5}} \operatorname{csch}^{-1}\left(\frac{\sqrt{2}}{\sqrt{5}}x\right) + c.$

(28) $\int \frac{dx}{\sqrt{e^{2x}-1}} \times \frac{e^x}{e^x} = \int \frac{e^x}{e^x\sqrt{(e^x)^2-1}} dx.$

$= \sec^{-1}(e^x) + c$

متطابقات الدوال المثلثية والزائدية:

م. عُولي بن ساسي

1. $\sin^2 x + \cos^2 x = 1$
2. $\sec^2 x = \tan^2 x + 1$
3. $\csc^2 x = \cot^2 x + 1$
4. $\sin^2 x = \frac{1}{2}(1 - \cos 2x)$
5. $\cos^2 x = \frac{1}{2}(1 + \cos 2x)$
6. $\sin x = 2 \sin \frac{x}{2} \cos \frac{x}{2}$
7. $\cos x = \cos^2 \frac{x}{2} - \sin^2 \frac{x}{2}$
8. $\sin x \sin y = \frac{1}{2}(\cos(x-y) - \cos(x+y))$
9. $\cos x \cos y = \frac{1}{2}(\cos(x-y) + \cos(x+y))$
10. $\sin x \cos y = \frac{1}{2}(\sin(x+y) + \sin(x-y))$
11. $\sin(x \pm y) = \sin x \cos y \pm \sin y \cos x$
12. $\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$

① $\cosh^2 x - \sinh^2 x = 1$

② $\operatorname{sech}^2 x = 1 - \tanh^2 x.$

③ $\operatorname{csch}^2 x = \coth^2 x - 1$

④ $\sinh^2 x = \frac{1}{2}((\cosh(2x) - 1).$

⑤ $\cosh^2 x = \frac{1}{2}(1 + \cosh(2x))$

⑥ $\sinh^2 x = 2\sinh\frac{x}{2}\cos\frac{x}{2}.$

⑦ $\cosh x = \cosh^2\frac{x}{2} + \sinh^2\frac{x}{2}$

⑧ $\sinh x \sinh y = \frac{1}{2}(\cosh(x+y) - \cosh(x-y))$

⑨ $\cosh x \cosh y = \frac{1}{2}(\cosh(x+y) + \cosh(x-y))$

⑩ $\sinh x \cosh y = \frac{1}{2}(\sinh(x+y) + \sinh(x-y))$

⑪ $\sinh(x \pm y) = \sinh x \cosh y \pm \sinh y \cosh x.$

⑫ $\cosh(x \pm y) = \cosh x \cosh y \pm \sinh x \cosh y.$