Control theory II
E.E.E. Department.

Midterm exam
(Fall 2024/2025)

Date 23/12/2024
Time: 1:30 H

Q1 (5 points). Given the differential equation that describes the motion of the pendulum illustrated in Figure 1:

$$m\,L^2\,\ddot{\theta} + b\,\dot{\theta} + m\,g\,L\,\theta = T_C$$

Where T_C is the input torque.

- Derive the state-space model of the pendulum.
- Discuss the controllability of the system.
- Design a full state feedback control that ensures the poles of the system are placed at $s_{1,2} = -1$.

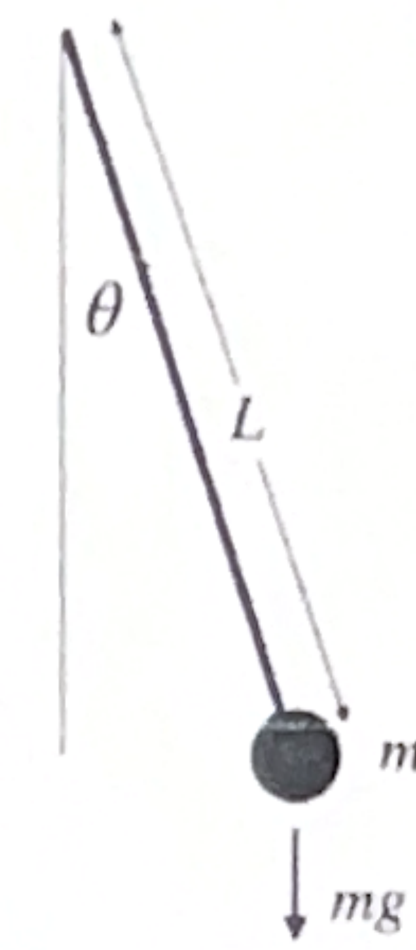

Figure (1)

Q2 (5 points). For the system described by the transfer function:

$$G(s) = \frac{Y(s)}{U(s)} = \frac{2\,s + 16}{s^3 + 9\,s^2 + 26\,s + 24}$$

- Derive the state-space model of the pendulum and represent it in a block diagram.
- Discuss the observability of the system.

Q3 (10 points (5+4)). Consider the state and output equations for a DC motor model given as:

$$\begin{bmatrix} \dot{i}_a \\ \dot{\omega} \end{bmatrix} = \begin{bmatrix} -100 & -5 \\ 5 & -10 \end{bmatrix} \begin{bmatrix} i_a \\ \omega \end{bmatrix} + \begin{bmatrix} 100 \\ 0 \end{bmatrix} v_a \,, \qquad y = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} i_a \\ \omega \end{bmatrix}$$

Part 1: Before adding the control loop.

- Find the characteristic equation of the system.
- Determine the location of the poles of this system.
- Is this system stable or not? Why?
- Is this system controllable or not? Why?
- Is this system observable or not? Why?

Part 2: Adding Feedback Control.

If we configure the system for state feedback control as in figure (2), find the value of the gain K that satisfies $\omega_n = 70.7107$ and $\zeta = 1.0607$.

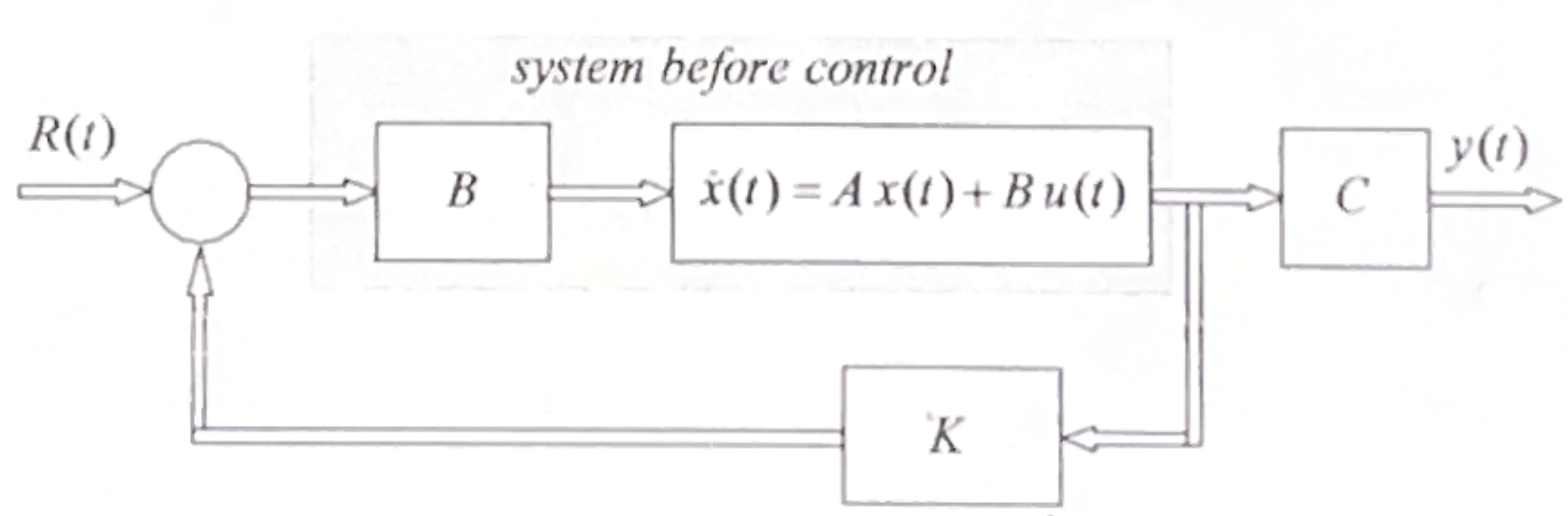

Figure (2)