Q1 *(12 point)*. For the control system of figure (1).

- Find the state and output equations.
- Check the controllability of the system.
- Check the observability of the system.
- Find the transfer function of the system.

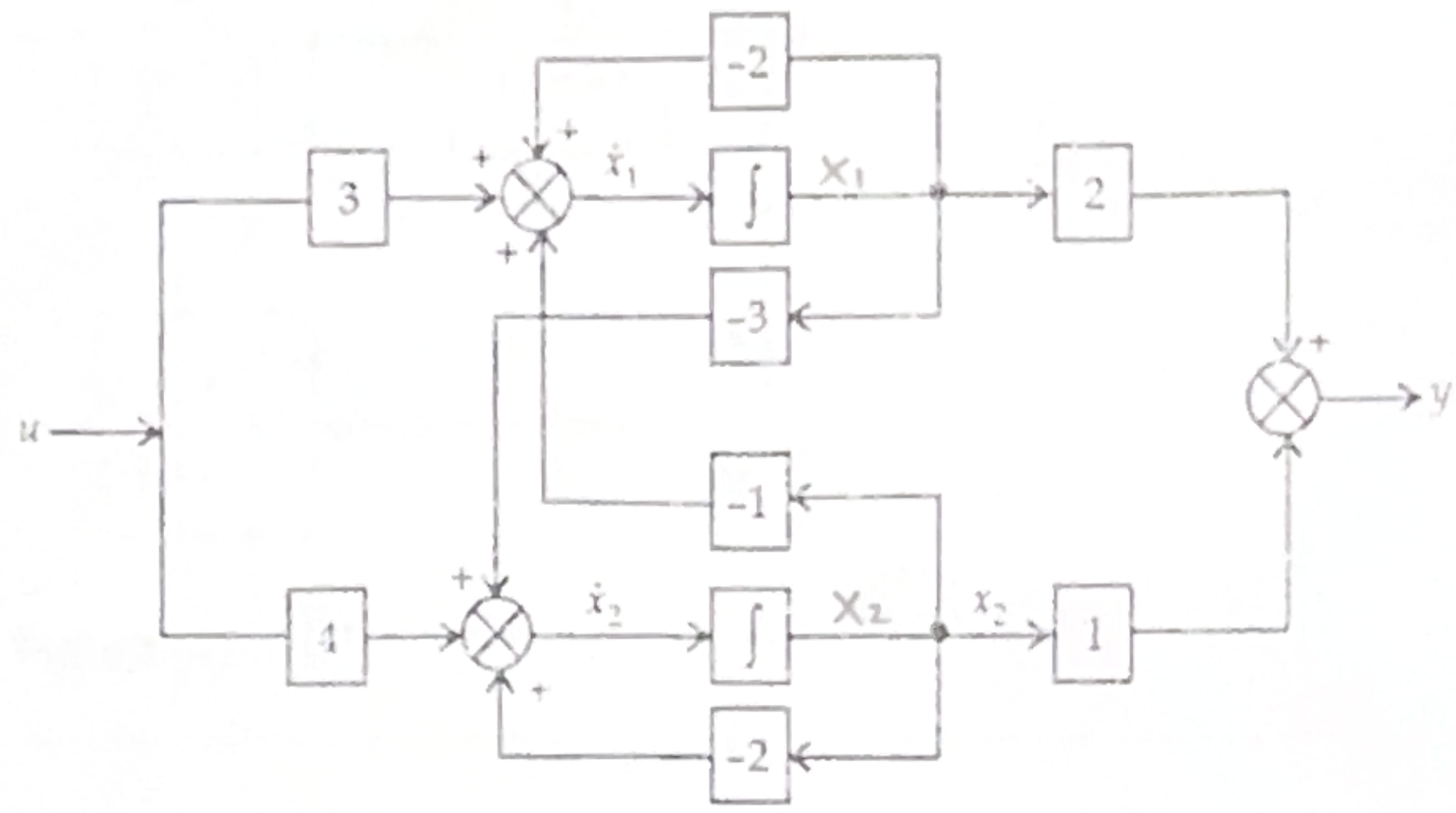

Figure (1)

Q2 (12 point). Consider the circuit in Figure (2), with an input voltage source $u(t)$ and an output current $y(t)$.

a) Using the capacitor voltage ($x_1 = v_c$) and inductor current ($x_2 = i_L$) as state variables, write state and output equations for the system.

b) Find the conditions relating R_1, R_2, C, and L that keeps the system controllable.

c) Find the conditions relating R_1, R_2, C, and L that keeps the system observable.

Note: $i_c = C\frac{dv_c}{dt}$ and $v_L = L\frac{di_L}{dt}$.

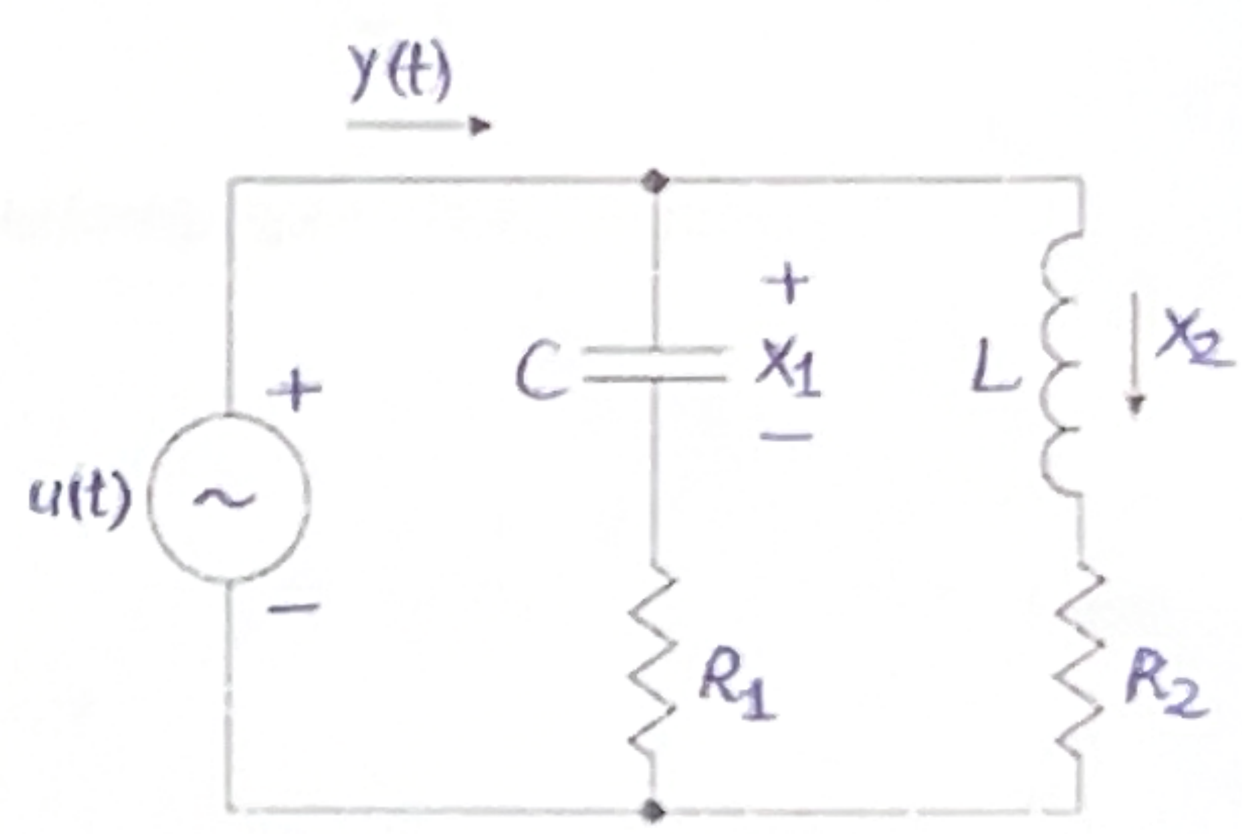

Figure (2)

Q3 (12 point). Suppose an undamped oscillator with state space description given by

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\lambda^2 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

- Find eigenvalues of the open loop system.

- Find the values of the undamped natural frequency ω_n and the damping ratio ζ of the open loop system.

- Find the control law (the value of K) that places the closed loop poles of the system so that they are both at $-2\omega_n$.

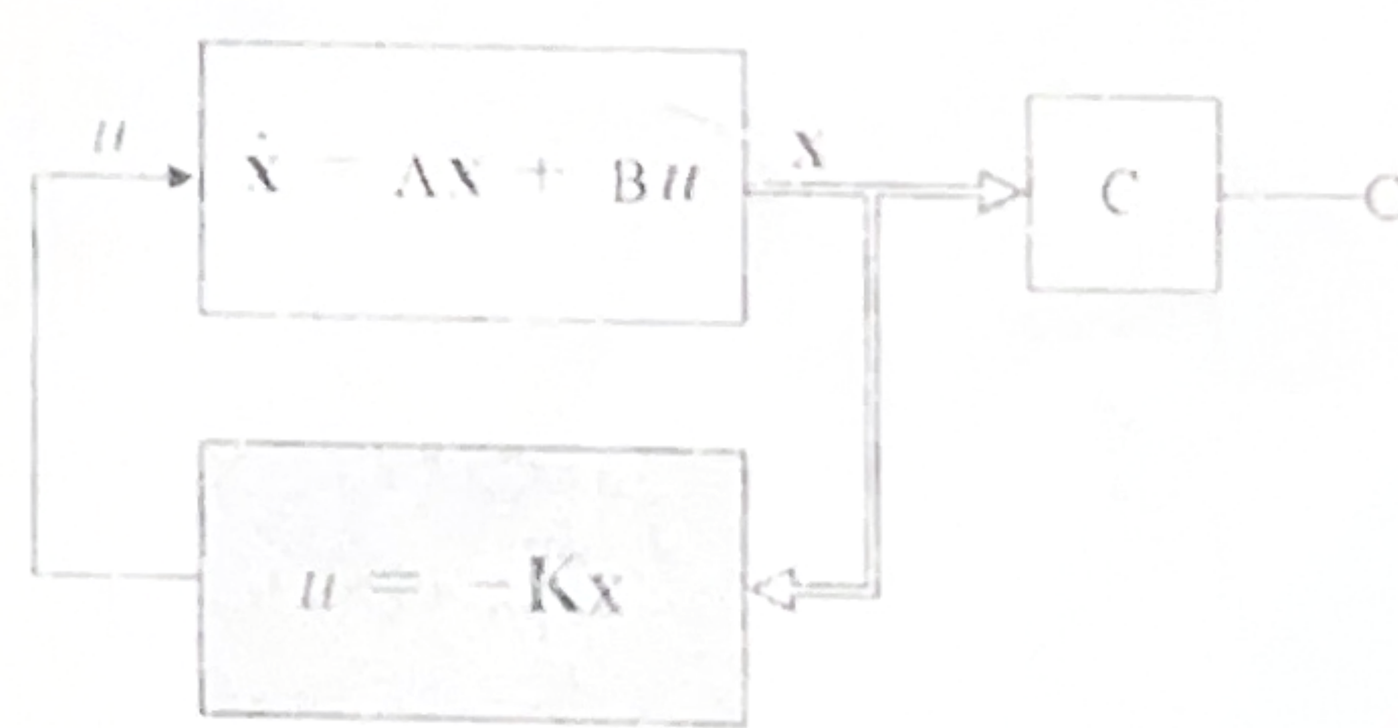

Figure (3)

Q4 (12 points). For the control system of figure (4):

- Discuss the possibility of obtaining a steady state error equal to 0.02 to a unit step input.
- Discuss the effect of adding a pole at $s = 0$.

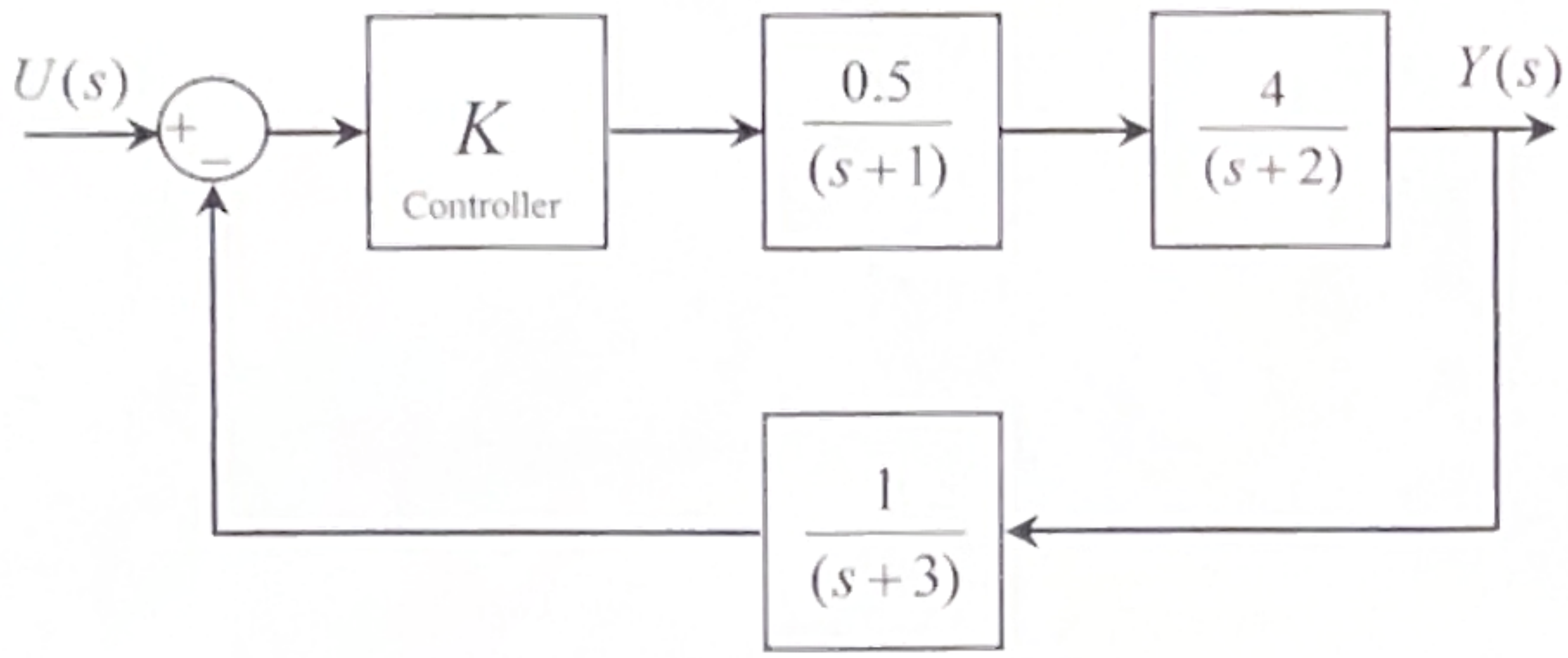

Figure (4)

Q5 (12 point). Consider the following system:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -2 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u \quad , \quad y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Determine the optimal state feedback gain matrix such that the following performance index is minimized:

$$J = \int_0^{\infty} (x^T Q\, x + u^T R\, u)\, dt$$

Where $Q = \begin{bmatrix} 100 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, $R = 0.01$, $P = \begin{bmatrix} p_1 & 14.6711 & 1 \\ 14.6711 & p_2 & 0.5312 \\ 1 & 0.5312 & p_3 \end{bmatrix}$.
