

Date: 19/12/2024

Mid Exam

Time: 90 min.

Problem 1 (6 points).

A random variable Y is related to another random variable X by $Y = aX + b$.

- Show that $\sigma_Y = a \sigma_X$.
- Find μ_Y and σ_Y if $Y = (X - \mu_X)/\sigma_X$.

Problem 2 (5 points).

A random process $X(t) = A$, in which A is a random variable has a mean value of 10 and a variance of 25. In the presence of independent noise having an autocorrelation function of $R_N(\tau) = 100e^{-10|\tau|}$

- Find the auto-correlation function of the sum of these two processes.
- Determine the value of τ at which the auto-correlation value of the sum is within 1% of its value at $\tau = \infty$.

Problem 3 (9 points).

- The channel power delay profile is given as 0 dB at $\tau = 0$, -6 dB at $\tau = 2 \mu s$, -12 dB at $\tau = 4 \mu s$, and -16 dB at $\tau = 7 \mu s$. Draw the channel power delay profile. Determine the average delay spread, rms delay spread, and maximum delay spread of the channel.
- Compare the average probability of bit error for BPSK modulation under no diversity, and two-branch MRC. Assume i.i.d. Rayleigh fading on each branch with equal branch SNR of 10 dB and of 20 dB. How does the relative performance change as the branch SNR increases.

Problem 4 (10 points).

- Find the critical distance d_c under the two-path model for a large macrocell in a suburban area with the base station mounted on a tower or building ($h_t = 20$ m), the receivers at height $h_r = 3$ m, and $f_c = 2$ GHz. Is this a good size for cell radius in a suburban macrocell? Why or why not?
- Given a transmitter which radiates a sinusoidal carrier frequency of 1850 MHz. For a vehicle moving at 30 m/sec, calculate the received carrier frequency if the mobile is moving (a) directly toward the transmitter (b) directly away from the transmitter (c) in a direction which is perpendicular to the direction of arrival of the transmitted signal.