

Date: 06/01/2025

Final Exam

Time: 3 Hours

Problem 1 (14 points).

a) Define a random process $Z(t) = X(t) + bX(t - 0.1)$, where $X(t)$ is a stationary random process whose autocorrelation function is $R_X(\tau) = 5\exp(-5|\tau|)$.

(1) Find $R_Z(\tau)$. (4 points)

(2) Find the value of b that minimizes the mean-square value of $Z(t)$. (3 points)

b) A Random process is described by

$$X(t) = A + B\cos(\omega t + \theta)$$

Where A is a random variable that is uniformly distributed between -5 and 5 , B is a Gaussian random variable with zero mean and a variance of 25 , ω is a constant, and θ is a random variable that is uniformly distributed from $-\pi/2$ to $3\pi/2$. A , B , and θ are statistically independent.

(1) Calculate the mean and variance of this process. (4 points)

(2) Is this process stationary in the wide sense? Why? (3 points)

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Problem 2 (12 points).

(a) Consider a cellular system operating at 900 MHz where propagation follows free space path loss with variations about this path from log-normal shadowing with $\sigma = 6$ dB. Suppose that for acceptable voice quality a signal-to-noise power ratio of 15 dB is required at the mobile. Assume the base station transmits at 1 W and its antenna has a 3 dB gain. There is no antenna gain at the mobile and the receiver noise in the bandwidth of interest is ⁻⁶⁰~~-60~~ dBm. Find the maximum cell size so that a mobile on the cell boundary will have acceptable voice quality 90% of the time. (Hint: $Q(-x) = 1 - Q(x)$)

(b) (a) Find the path loss under the Hata model assuming $f_c = 900$ MHz, $h_t = 20$ m, $h_r = 5$ m and $d = 100$ m for a large urban city, a small urban city. Explain qualitatively the path loss differences for these 2 environments.

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Problem 3 (8 points).

Consider a fading distribution $f_\gamma(\gamma)$ where $\int_0^\infty f_\gamma(\gamma) e^{-x\gamma} d\gamma = \frac{0.01 \text{ SNR}}{\sqrt{x}}$ and $\gamma = |h_i|^2$. Find the average probability of bit error (Average BER) for a BPSK modulated signal where the

receiver has 2-branch diversity with MRC combining, and each branch has an average SNR of 10 dB and experiences independent fading with distribution $f_\gamma(\gamma)$.

(Use fact that: $Q(z) = \int_z^\infty \frac{1}{\sqrt{2\pi}} e^{-\frac{y^2}{2}} dy = \frac{1}{\pi} \int_0^{\pi/2} e^{-\frac{z^2}{2(\sin\phi)^2}} d\phi$)

Problem 4 (14 points).

- Name a class of codes belongs to Orthogonal Space-Time Block Codes (OSTBC).
- Illustrate in brief the advantages of using MIMO technique
- For the MIMO system, suppose CSI is known at both transmitter and receiver. If we are willing to decrease the rate at the transmitter, which technique is used at the transmitter to get much less value of BER ? (3 points)
- Consider a 1x2 MISO system with channel gain vector $\mathbf{h} = [1+j \quad 3+4j]$ is known at the receiver, and there is a total transmit power of $P_t = 10$ mW across the two transmit antennas, AWGN with power $N_0 = 10^{-9}$ W/Hz at the receive antenna, and bandwidth BW = 100 KHz. Determine the SNR of each symbol at the receiver resulting from using Alamouti code method. (6 points)

Problem 5 (12 points).

- Relating to information transfer in communication links, explain in brief the capacity and entropy. (4 points)
- Consider a 2x2 MIMO system with channel gain matrix $\mathbf{H}$ given by

$$\mathbf{H} = \begin{bmatrix} 0.3 & 0.5 \\ 0.7 & 0.2 \end{bmatrix}$$

Assume $\mathbf{H}$ is known at the receiver, and that there is a total transmit power of $P_t = 10$ mW across the two transmit antennas, AWGN with power $N_0 = 10^{-9}$ W/Hz at each receive antenna, and bandwidth BW = 100 KHz. Find

- The data rate per unit BW.
- The capacity of this 100 KHz bandlimited channel. (8 points)

Good Luck