إجابة الامتحان النهائي ربيع 2026

① $\int x^6\left(\frac{1}{x}+\frac{1}{x^2}\right)^3 dx.$

$\int (x^2)^3\left(\frac{1}{x}+\frac{1}{x^2}\right) dx.$

$\int \left(x^2\left(\frac{1}{x}+\frac{1}{x^2}\right)\right)^3 dx.$

$\int (x+1)^3\, dx.$

$I = \frac{1}{4}(x+1)^4 + C$

② $\int \frac{8^{1+x}+4^{1+x}}{2^{2x}} dx.$

$\int \frac{(2^3)^{1+x}+(2^2)^{1+x}}{2^{2x}} dx$

$\int \frac{2^{3+3x}+2^{2+2x}}{2^{2x}} dx$

$\int \frac{2^3 . 2^{3x}+2^2 . 2^{2x}}{2^{2x}} dx$

 يتبع

$$\int \left[\frac{2^3 \cdot \cancel{2^{3x}}^{\,2^x}}{\cancel{2^{2x}}} + \frac{2^2 \cdot \cancel{2^{2x}}}{\cancel{2^{2x}}} \right] dx.$$

$$\int \left(2^3 \cdot 2^x + 2^2 \right) dx$$

$$4\int \left(2\,2^x + 1 \right) dx.$$

$$I = 4\left[\frac{2\,2^x}{\ln 2} + x \right] + C$$

(3) $$\int \frac{\sin^2 x}{(1+\cos x)^2} \cdot dx, \rightarrow \int \frac{1-\cos^2 x}{(1+\cos^2 x)^2}\, dx.$$

$$\int \frac{(1+\cos x)(1-\cos x)}{(1+\cos^2 x)^2}\, dx.$$

$$\int \frac{1-\cos x}{1+\cos x}\, dx. \rightarrow \int \frac{2\sin^2\left(\frac{x}{2}\right)}{2\cos^2\left(\frac{x}{2}\right)}\, dx.$$

$$\int \tan^2\left(\frac{x}{2}\right) dx. \rightarrow \int \left(\sec^2\left(\frac{x}{2}\right) - 1\right) dx$$

$$I = 2\tan\left(\frac{x}{2}\right) - x + C$$

④ $\int \frac{-dx}{\sqrt{1-x-x^2}}$

$$\int \frac{-dx}{\sqrt{-(x^2+x-1}} \longrightarrow \int \frac{-dx}{\sqrt{-(x^2+x+\frac{1}{4}-\frac{1}{4}-1)}}$$

$$\int \frac{-dx}{\sqrt{-((x^2+x+\frac{1}{4})-\frac{5}{4}}} \longrightarrow \int \frac{-dx}{\sqrt{-((x+1/2)^2-\frac{5}{4})}}$$

$$\int \frac{-dx}{\sqrt{\frac{5}{4}-(x+1/2)^2}}$$

$$x+1/2 = \frac{\sqrt{5}}{2}\sin\theta \longrightarrow dx = \frac{\sqrt{5}}{2}\cos\theta \, d\theta$$

$$\int \frac{-1}{\sqrt{5/4-5/4\sin^2\theta}} \cdot \frac{\sqrt{5}}{2}\cos\theta \, d\theta$$

$$\int \frac{-d\theta}{\frac{\sqrt{5}}{\cancel{2}}\cancel{\cos\theta}} \cdot \frac{\sqrt{5}}{2}\cancel{\cos}\theta$$

$$\int -d\theta \longrightarrow -\theta + C$$

$$I = \sin^{-1}\left(\frac{2}{\sqrt{5}}(x+1)\right)+C$$

③

⑤ $\displaystyle\int \frac{1}{3\sin x - 4\cos x}\,dx.$

$$\int \frac{1}{3\left(\frac{2z}{1+z^2}\right) - 4\left(\frac{1-z^2}{1+z^2}\right)} \cdot \frac{2\,dz}{1+z^2}$$

$$\int \frac{2\,dz}{6z - 4 + 4z^2} \rightarrow \frac{2}{2}\int \frac{dz}{2z^2+6z-4}$$

$$\int \frac{dz}{(z+2(2z-1)} \longrightarrow \frac{A}{z+2} + \frac{B}{2z+1}$$

$$1 = 2zA + A + Bz + 2B.$$

$$A + 2B = 1$$
$$2A + B = 0 \quad \longmapsto \quad \boxed{A = -1/3},\ \boxed{B = 2/3}$$

$$\int \left[\frac{-1/3}{z+2} + \frac{2/3}{2z+1}\right] dz.$$

$$I = -\tfrac{1}{3}\ln(z+2) + \tfrac{1}{3}\ln(2z+1) + C$$

⑥ $\int \frac{1}{x^2-b^2}\left(\frac{x+b}{x-b}\right)^3 dx.$

$$u=\frac{x+b}{x-b} \longrightarrow du=\frac{\overbrace{(\cancel{x}-b)-(\cancel{x}+b)}^{-2b}}{(x-b)^2}.dx.$$

$$du=\frac{-2b}{(x-b)^2}\cdot dx \rightarrow dx=\frac{(x-b)^2}{-2b}du$$

$$I=\int\frac{1}{(x\cancel{-b})(x+b)}(u)^3\frac{(x-b)^{\cancel{2}}}{-2b}\ du.$$

$$I=-\frac{1}{2b}\int u^{\cancel{3}\,2}\cdot\frac{1}{u}\ du.$$

$$\frac{-1}{2b}\int u^2\ du \longrightarrow I=\frac{-1}{2b}\cdot\frac{u^3}{3}+C.$$

$$I=\frac{1}{-6b}\left(\frac{x+b}{x-b}\right)^3+C$$

7 $\int e^{2x} \cdot \ln(2+e^x) \cdot dx \longrightarrow \int (e^x)^2 \cdot \ln(2+e^x)\, dx$

$u = 2+e^x \longrightarrow du = e^x\, dx \longrightarrow du = \frac{dx}{e^x}$

$e^x = u-2 \qquad \longrightarrow du = \frac{du}{u-2}$

$$I = \int (u-2)^2 \ln u \cdot \frac{du}{u \cancel{-2}} \longrightarrow \int (u-2) \ln u\, du$$

$z = \ln u \longrightarrow dv = (u-2)$

$dz = \frac{1}{u} du \longrightarrow v = \frac{u^2}{2} - 2u$

$$I = \left(\frac{u^2}{2} - 2u\right) \ln u - \int \frac{1}{u}\left(\frac{u^2}{2} - 2u\right) du$$

$$I = \left(\frac{u^2}{2} - 2u\right) \ln u - \int \frac{u^2 - 4u}{2u}\, du$$

$$I = \left(\frac{u^2}{2} - 2u\right) \ln u - \int \frac{\cancel{u}(u-4)}{\cancel{u}}\, du$$

$$I = \left(\frac{u^2}{2} - 2u\right) \ln u - \frac{1}{2}\left(\frac{1}{2}u^2 - 4u\right) + C$$

$$I = \left[\frac{(2+e^x)^2}{2} - 2(2+e^x)\right] \ln|2+e^x| - \frac{(2+e^x)^2}{4} + 4(2+e^x) + C$$

6

Q2: (1)

$$\int_0^{\pi/4} \frac{1}{(\sec x + \tan x)^2}\, dx, \quad \rightarrow \sec^2 x - \tan^2 x = 1$$

$$\int_0^{\pi/4} \frac{\sec^2 x - \tan^2 x}{(\sec x + \tan x)^2}\, dx \longrightarrow \int_0^{\pi/4} (\sec x - \tan x)^2.$$

$$\int_0^{\pi/4} (\sec^2 x - 2\sec x \tan x + \tan^2 x)\, dx.$$

$$\int_0^{\pi/4} (\sec^2 x - 2\sec x \tan x + \sec^2 x - 1)\, dx.$$

$$\int_0^{\pi/4} (2\sec^2 x - 1 - 2\sec x \tan x)\, dx.$$

$$I = 2\tan x - x - 2\sec x \Big|_0^{\pi/4}$$

$$I = \left(2 - \frac{\pi}{4} - 2\sqrt{2}\right) - (0 - 0 - 2)$$

$$I = 4 - 2\sqrt{2} - \frac{\pi}{4}$$

② $\int_0^2 \left| |x-1| - x \right| dx \rightarrow |x-1| = 0, \boxed{x=1}$

$$\int_0^1 \left| -(x-1) - x \right| dx + \int_1^2 \left| x-1-x \right| dx.$$

$$\int_0^1 |1-2x|\, dx + \int_1^2 dx \rightarrow |1-2x| = 0, \boxed{x = \frac{1}{2}}$$

$$\int_0^{1/2} (1-2x)\, dx + \int_{1/2}^1 (2x-1)\, dx + \int_1^2 dx.$$

$$I = (x - x^2)\Big|_0^{1/2} + (x^2 - x)\Big|_{1/2}^1 + x\Big|_1^2.$$

$I = 3/2.$

Q3

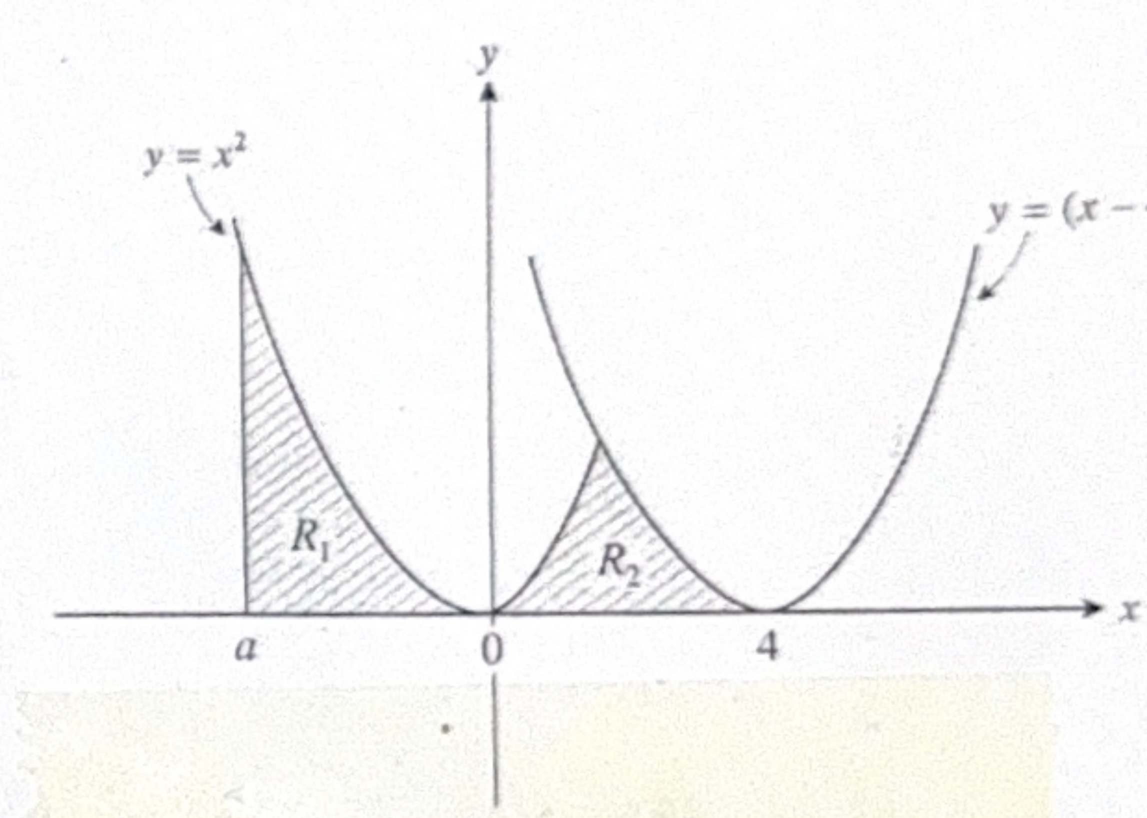
(1) $\int \frac{\cos x}{\sin^3 x - \sin^2 x} \, dx.$

$$\int \frac{\cos x}{\sin x^2 (\sin x - 1)} \, dx.$$

$u = \sin x \longrightarrow du = \cos x \, dx.$

$$\int \frac{du}{u^2(u-1)} \longrightarrow \int \frac{A}{u} + \frac{B}{u^2} + \frac{C}{u-1}$$

(2) $y = x^2$, $y = (x-4)^2$



$x^2 = x^2 - 8x + 16 \rightarrow \boxed{x = 2}$

$R_1 = \frac{1}{2} R_2 \rightarrow R_2 = 2R_1$

$$\int_0^2 x^2 \, dx + \int_2^4 (x^2 - 8x + 16) \, dx = 2\int_a^0 x^2 \, dx.$$

$$I = \frac{1}{3}x^3 \Big|_0^2 + \frac{1}{3}x^3 - 4x^2 + 16x \Big|_2^4 + \frac{2}{3}x^3 \Big|_a^0$$

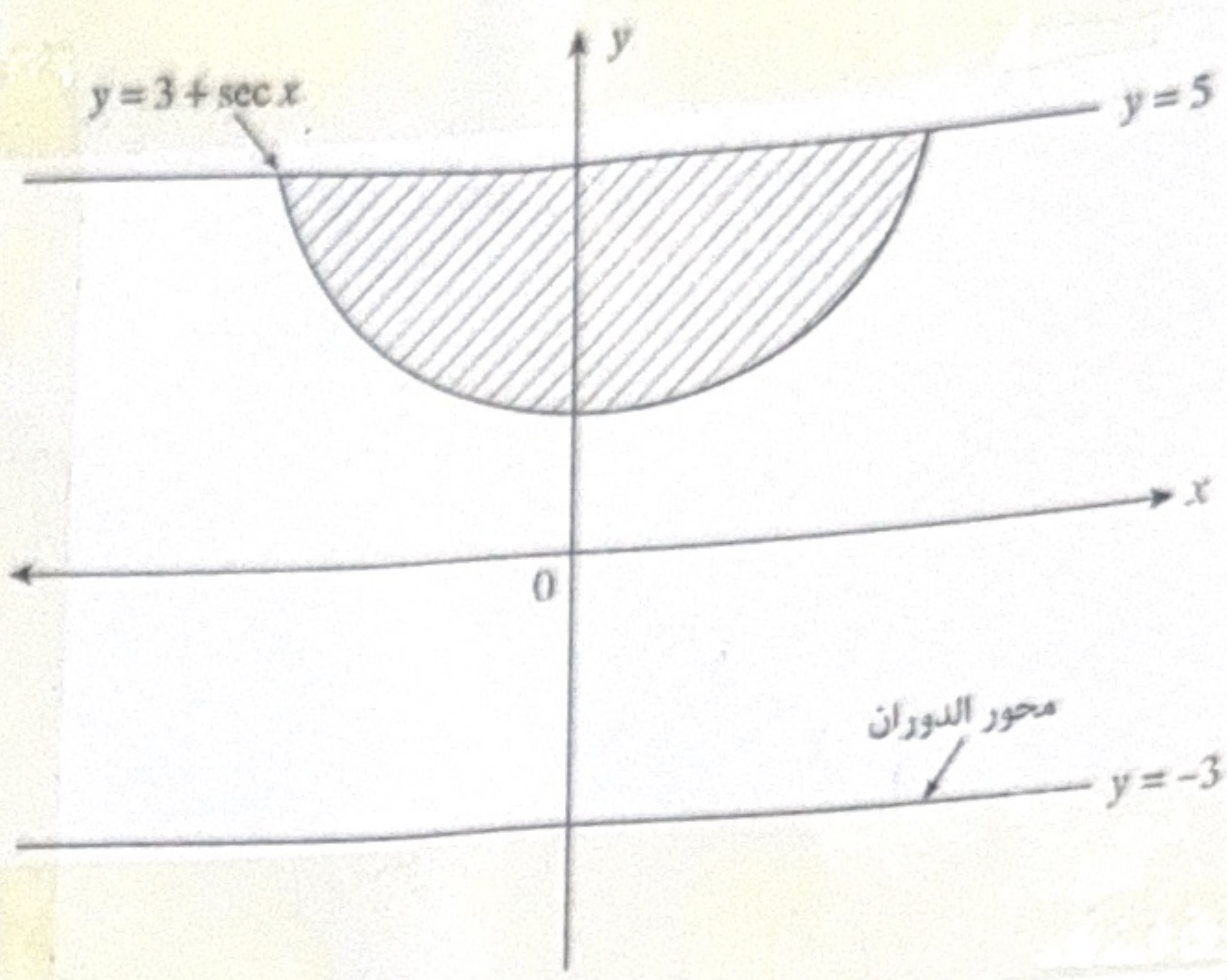
$a^3 = -8 \rightarrow \boxed{a = -2}$

Q4:

محور الدوران (c)
لأنه في الأسفل

$$V = \pi \int_a^b (y_1 - c)^2 - (y_2 - c)\, dx$$

$$V = \pi \int_{-\pi/3}^{\pi/3} (5-(-3))^2 - (3+\sec x - (-3))\, dx.$$



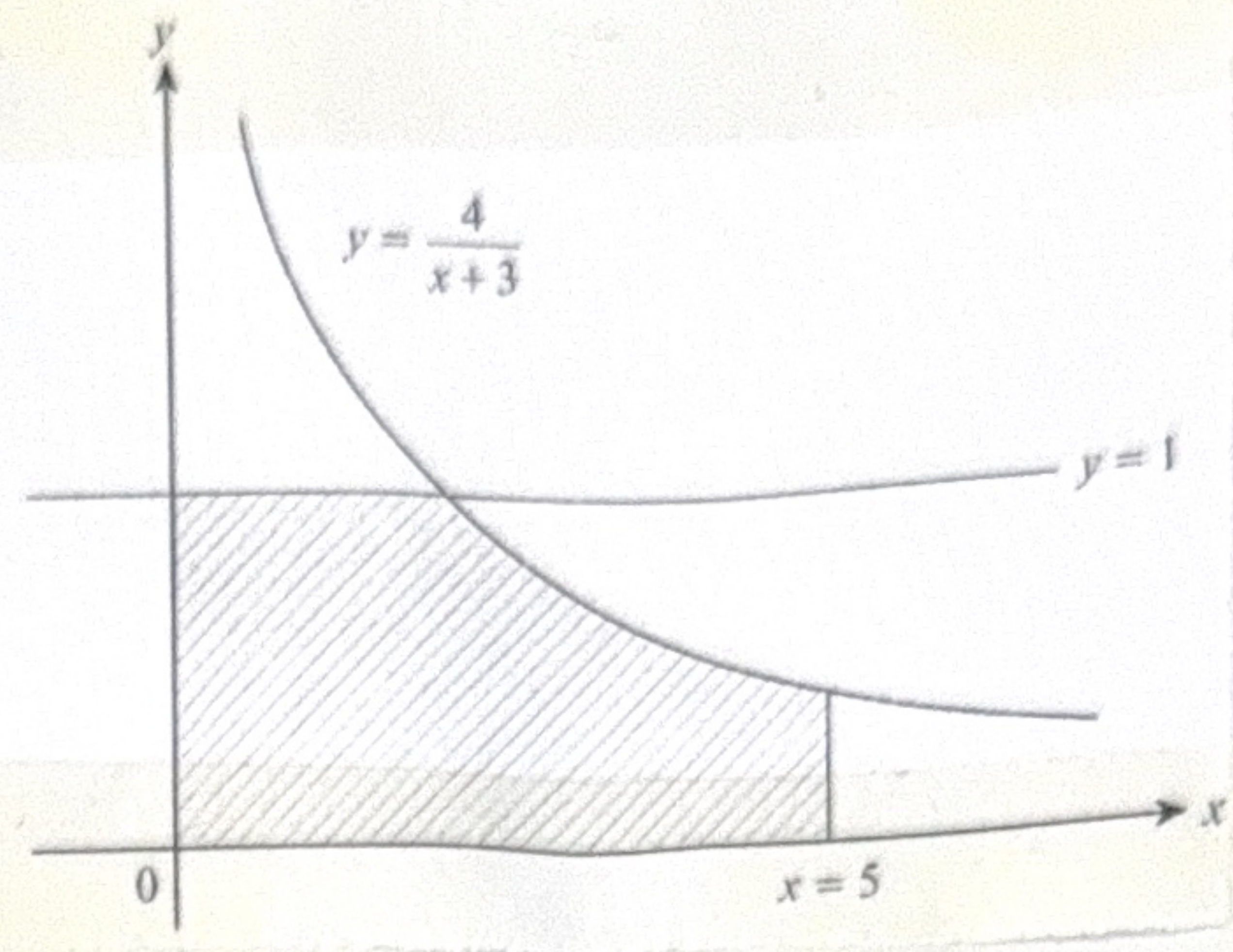
$$V = \pi \int_{-\pi/3}^{\pi/3} 8^2 - (6+\sec x)^2\, dx.$$

$$V = \pi \int_{-\pi/3}^{\pi/3} 64 - (36 + 12\sec x + \sec^2 x)\, dx.$$

$$V = \pi \int_{-\pi/3}^{\pi/3} (28 - 12\sec x - \sec^2 x)\, dx.$$

Q5:

① $y = 1 \quad , y = \dfrac{4}{x+3}.$



$$1 = \frac{4}{x+3} \rightarrow x = 1$$

$$A_{1\,\square} = 1 \times 1 = \boxed{1}$$

$$A_2 = \int_1^5 \frac{4}{x+3}\, dx.$$

$$= 4\ln|x+3| \Big|_1^5 \Longrightarrow 4(\ln(5+3)) - 4\ln|1+3|$$

$$A_2 = 4(\ln 8 - \ln 4)$$

$$A_2 = 4\left(\ln \frac{8}{4}\right) \rightarrow 4\ln 2$$

$$A_2 = \ln 2^4 = \ln 16$$

$$A_T = A_1 + A_2 = 1 + \ln 16$$

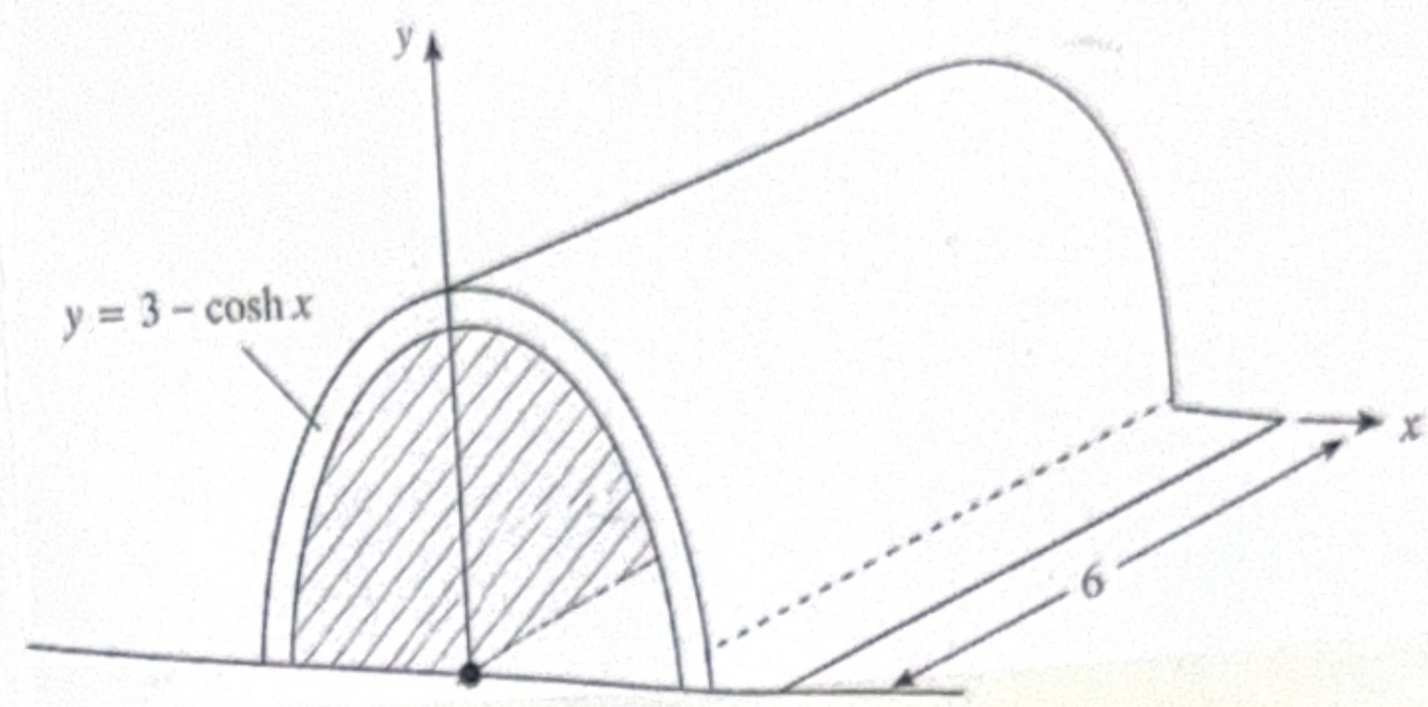
$$\boxed{A_T = 1 + \ln 16}$$

Q_5: $y = 3 - \cosh x$, $y = 0$.

العمق = $6m$

$x = \cosh^{-1} 3$, $x = -\cosh^{-1} 3$.

$$A = \int_{-\cosh 3}^{+\cosh 3} (3 - \cosh x)\, dx$$



$$A = (3x - \sinh x)\Big|_{-\cosh 3}^{+\cosh 3}$$

$$A = (3\cosh^{-1} 3 - \sinh(\cosh^{-1} 3)) - (-3\cosh^{-1} 3 - \sinh(-\cosh^{-1} 3))$$

$$V = 6(3\cosh^{-1} 3 - \sinh(\cosh^{-1} 3)) + (3\cosh^{-1} 3(-\cosh^{-1} 3)).\ m^3$$