Chapter # 1

Load and Stress Analysis

Concept of stress:

The main objective of this course (Mechanics of Materials) is to provide the future engineer with the means of analyzing and designing various machines and load structures.

Normal Stress: Axial Loading

The resultant of the internal forces for an axially loaded member is normal to a section cut perpendicular to a section axis.

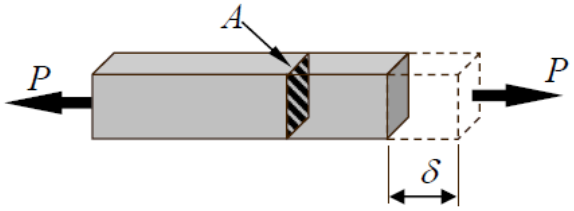
The force intensity on that section defined as the normal stress

- ✓ Direct Tension Normal Stress

$$\sigma_{ave} = \frac{P}{A}$$

- ✓ Axial Deformation

$$\delta = \frac{PL}{EA}$$



where:

E, Modulus of Elasticity

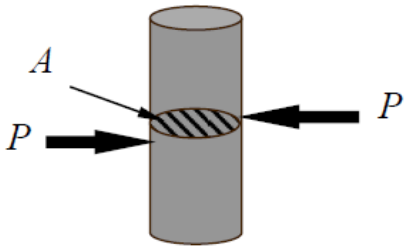
A, Cross-Sectional

L, Length of structural component

Shearing Stress

- ✓ Direct Shear Stress

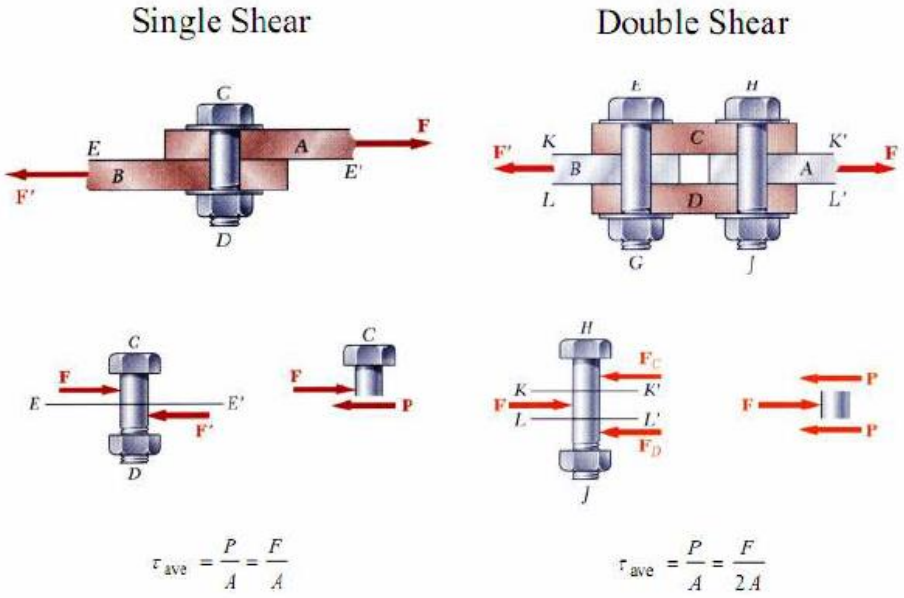
$$\tau_{ave} = \frac{P}{A}$$



Shearing Stress Examples:

The forces F and F' are applied transversely to the member AB. Corresponding internal forces act on the plane of section C and are called Shearing forces and the corresponding average shear stress is

$$\tau_{ave} = \frac{P}{A}$$



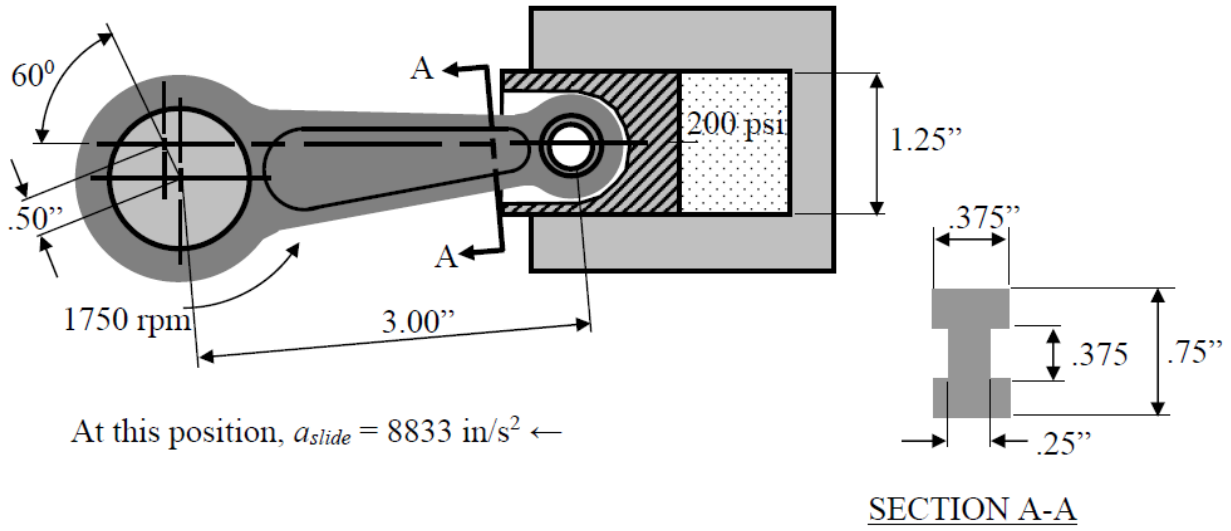
Physical Constants of Materials:

Material	Modulus of Elasticity, E (psi)	Modulus of Rigidity, G (psi)	Poisson's Ratio, ν	Unit Weight, w (lb/in^3)
Aluminum	10.4 x 10^6	3.9 x 10^6	0.333	0.098
Brass	15.4 x 10^6	5.8 x 10^6	0.324	0.309
Carbon Steel	30.0 x 10^6	11.5 x 10^6	0.292	0.282
Cast Iron	14.5 x 10^6	6.0 x 10^6	0.211	0.260
Copper	17.2 x 10^6	6.5 x 10^6	0.326	0.322
Nylon 6/6	12.0 x 10^3	4.2 x 10^3	0.425	0.041
Stainless Stl.	27.6 x 10^6	10.6 x 10^6	0.305	0.280
Zinc (Zamak 3)	12.4 x 10^6	4.7 x 10^6	0.325	0.257

Example 1-1:

For the loading on the slider-crank mechanism shown, determine the stress at the section indicated?

The piston weighs 0.5 lb

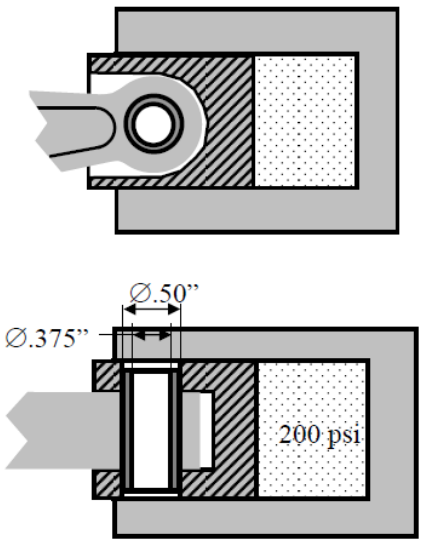


At this position, a_{slide} = 8833 in/s^2 ←

SECTION A-A

Example 1-2:

For the slider-crank mechanism previously discussed, determine the shear stress on the wrist pin.



Example 1-3:

Concept Application 1.1

Considering the structure of Fig. 1.1 on page 5, assume that rod *BC* is made of a steel with a maximum allowable stress $\sigma_{all} = 165$ MPa. Can rod *BC* safely support the load to which it will be subjected? The magnitude of the force F_{BC} in the rod was 50 kN. Recalling that the diameter of the rod is 20 mm, use Eq. (1.5) to determine the stress created in the rod by the given loading.

$$P = F_{BC} = +50\,\text{kN} = +50 \times 10^3\,\text{N}$$

$$A = \pi r^2 = \pi\left(\frac{20\,\text{mm}}{2}\right)^2 = \pi(10 \times 10^{-3}\,\text{m})^2 = 314 \times 10^{-6}\,\text{m}^2$$

$$\sigma = \frac{P}{A} = \frac{+50 \times 10^3\,\text{N}}{314 \times 10^{-6}\,\text{m}^2} = +159 \times 10^6\,\text{Pa} = +159\,\text{MPa}$$

Since σ is smaller than σ_{all} of the allowable stress in the steel used, rod *BC* can safely support the load.

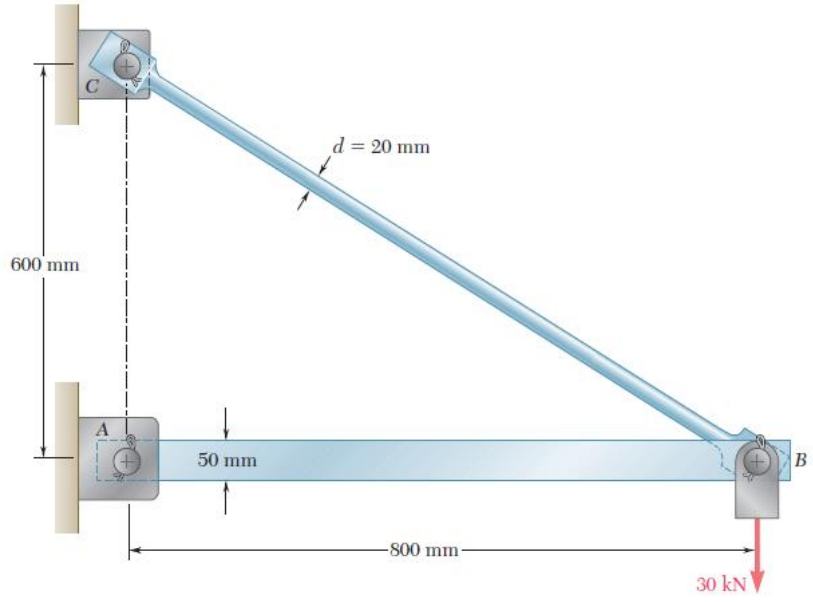


Fig. 1.1 Boom used to support a 30-kN load.

Example 1-4:

Concept Application 1.2

As an example of design, let us return to the structure of Fig. 1.1 on page 5 and assume that aluminum with an allowable stress $\sigma_{all} = 100$ MPa is to be used. Since the force in rod BC is still $P = F_{BC} = 50$ kN under the given loading, from Eq. (1.5), we have

$$\sigma_{all} = \frac{P}{A} \qquad A = \frac{P}{\sigma_{all}} = \frac{50 \times 10^3\,\text{N}}{100 \times 10^6\,\text{Pa}} = 500 \times 10^{-6}\,\text{m}^2$$

and since $A = \pi r^2$,

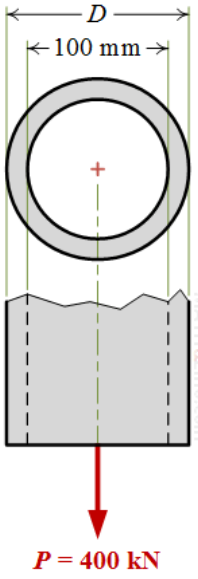
$$r = \sqrt{\frac{A}{\pi}} = \sqrt{\frac{500 \times 10^{-6}\,\text{m}^2}{\pi}} = 12.62 \times 10^{-3}\,\text{m} = 12.62\,\text{mm}$$

$$d = 2r = 25.2\,\text{mm}$$

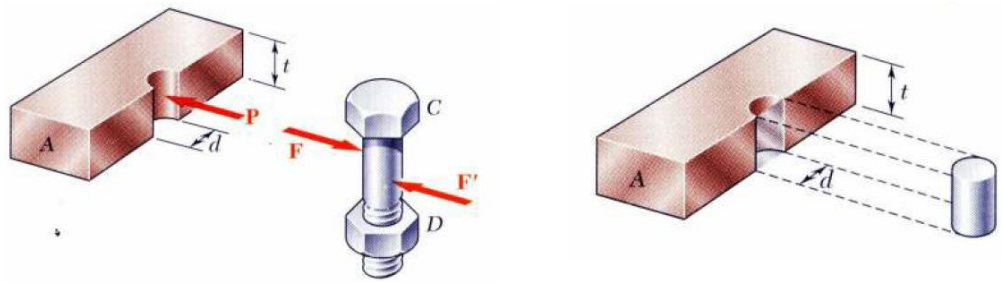
Therefore, an aluminum rod 26 mm or more in diameter will be adequate.

Example 1-5:

A hollow steel tube with an inside diameter of 100 mm must carry a tensile load of 400 kN. Determine the outside diameter of the tube if the stress is limited to 120 MN/m^2.



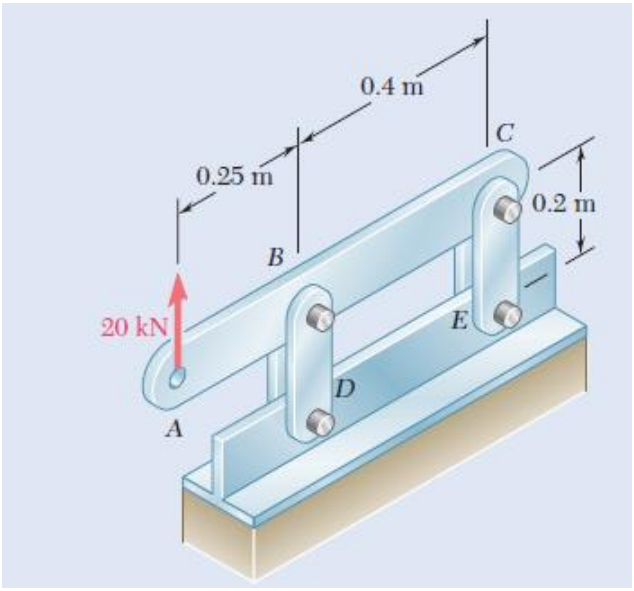
Bearing Stress in Connections:



- Bolts, pins, and rivets create stresses in the members they connect along the *bearing surface* or surface of contact.
- The resultant of the force distribution on the surface is equal and opposite to the force excreted on the pin
- Corresponding average force intensity is called the bearing stress

$$\sigma_b = \frac{P}{A} = \frac{P}{t\,d}$$

Example 1-6:



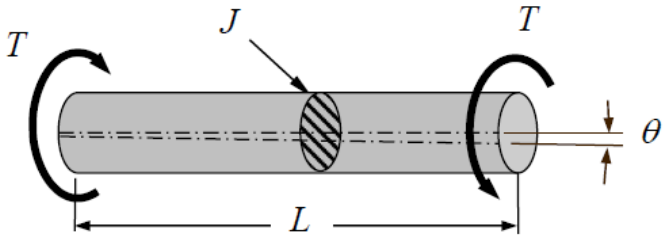
Each of the four vertical links has an 8*36-mm uniform rectangular cross section, and each of the four pins has a 16-mm diameter.

Determine (a) the average shearing stress at point C, (b) bearing stress in link CE at point C, (c) bearing stress in link ABC at point C, (d) the maximum value of the average normal stress in the links connecting points *B* and *D*

Torsional Loading

✓ Torsional Stress

$$\tau = \frac{Tr}{J}$$



✓ Torsional Deformation

$$\theta = \frac{TL}{GJ}$$

where

- r, radius to point of analysis (usually on the outer surface)
- J, polar moment of inertia (Charted for different shapes)
- G, Shear Modulus

Section Properties

Circular Section:

$$A = \frac{\pi D^2}{4} \qquad J = \frac{\pi D^4}{32} \qquad I = \frac{\pi D^4}{64}$$

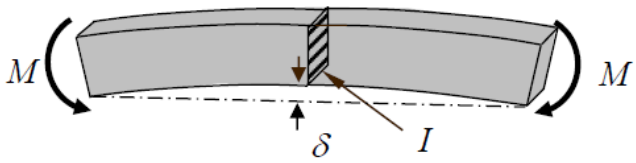
Hollow Circular Section:

$$A = \frac{\pi\left(D^2 - d^2\right)}{4} \qquad J = \frac{\pi\left(D^2 - d^2\right)}{32} \qquad I = \frac{\pi\left(D^2 - d^2\right)}{64}$$

Rectangular Section:

$$A = BH \qquad I = \frac{BH^3}{12}$$

Bending Loads



✓ Bending Normal Stress

$$\sigma = \frac{Mc}{I}$$

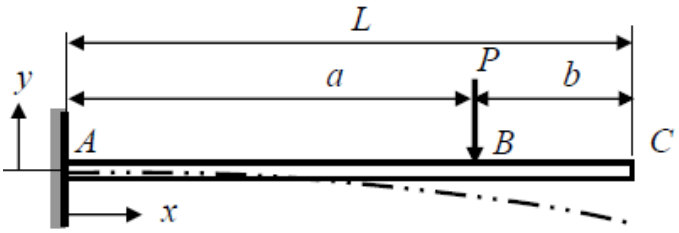
Where

- c, distance from neutral axis to point of analysis, usually on the outer surface.
- I, area moment of inertia (charted for different shapes)

✓ Flexural Deformation

$$\delta = \iint \frac{M}{EI} dl$$ (charted for different supports and load cases)

Cantilever

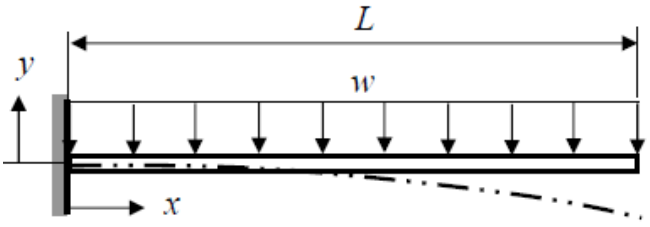


between A and B:

$$y = \frac{-Px^2}{6EI}(3a - x)$$

between B and C:

$$y = \frac{-Pa^2}{6EI}(3x - a)$$

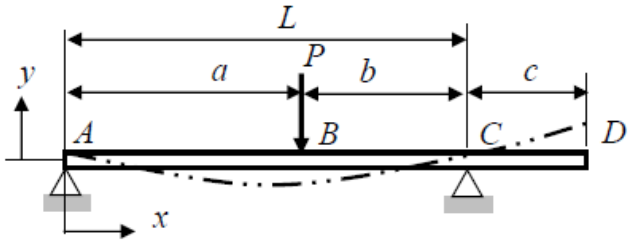


$$y = \frac{-wLx^2}{24EIL}\left[2L^2 + (2L - x)^2\right]$$

where:

w = load per unit length

Simple Support



between A and B:

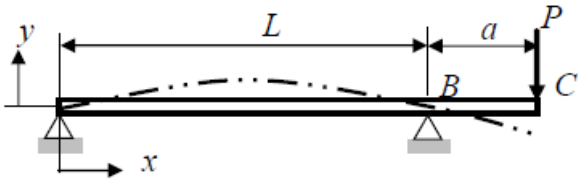
$$y = \frac{-Pbx}{6EIL}(L^2 - b^2 - x^2)$$

between B and C:

$$y = \frac{-Pa(L-x)}{6EIL}\left[2Lx - x^2 - a^2\right]$$

between C and D:

$$y = \frac{Pab(x-L)}{6EIL}(L+a)$$



between A and B:

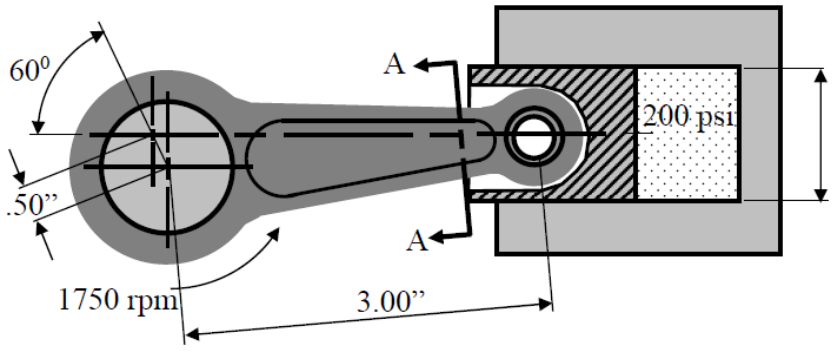
$$y = \frac{Pax}{6EIL}(L^2 - x^2)$$

between B and C:

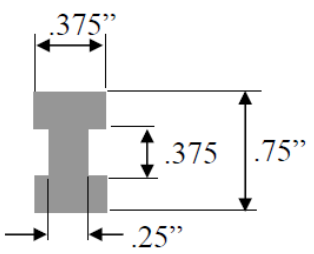
$$y = \frac{P(x-L)}{6EI}\left[(x-L)^2 - a(3x-L)\right]$$

Example 1-7:

For the slider-crank mechanism previously discussed and shown below, determine the torsional shear stress on the motor shaft.



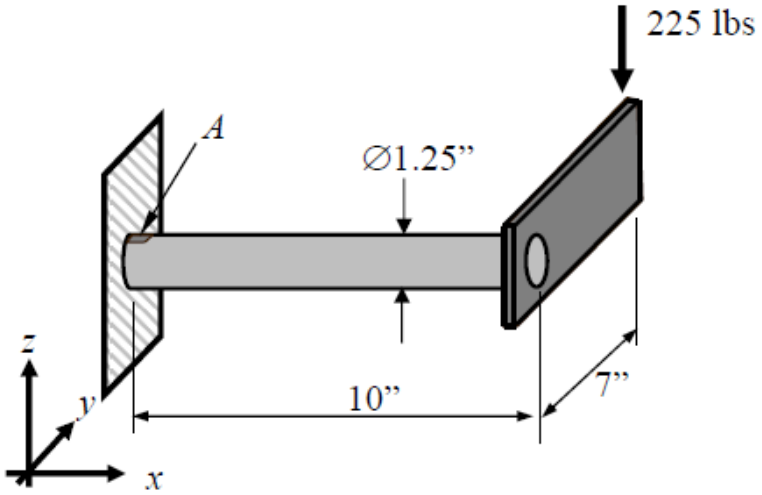
At this position, a_{slide} = 8833 in/s^2 ←



SECTION A-A

Example 1-8:

Calculate the stress components at point *A* of the shaft shown.

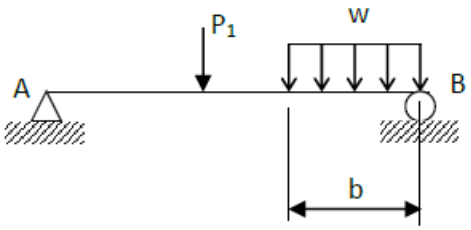


Supporters and Reactions:

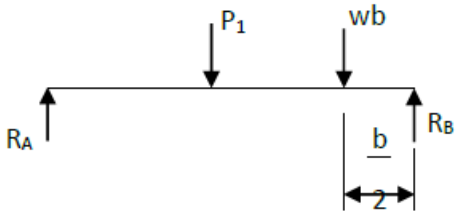
Type of connection	Reaction	Type of connection	Reaction
Cable	One unknown : F	External pin	Two unknowns : F_x , F_y
Roller	One unknown : F	Internal pin	Two unknowns : F_x , F_y
Smooth support	One unknown : F	Fixed support	Three unknowns : F_x , F_y , M

Converting Distributed Loads to Concentric Loads:

SIMPLY SUPPORT BEAM



FREE BODY DIAGRAM

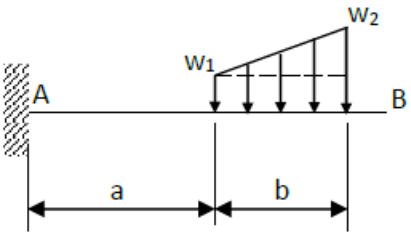


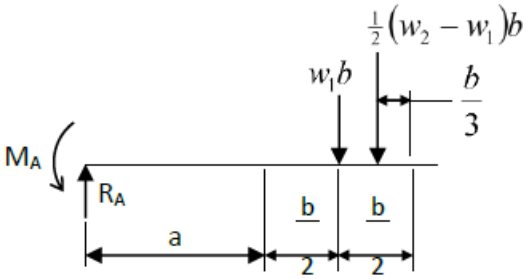
LOAD TYPES:

P_1 = concentrated load {force}

w = uniform load {force / distance}
(uniformly distributed)

CANTILEVER BEAM

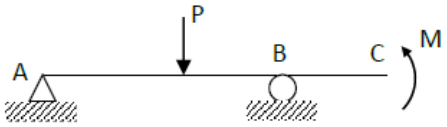


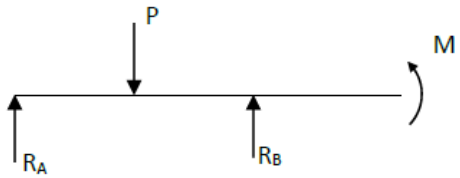


LOAD TYPES:
$w_1 - w_2$ = linearly varying load {force / distance}

$\frac{1}{2}(w_2 - w_1)b$ acts at centroid (i.e., $\frac{b}{3}$) of geometry

BEAM WITH OVERHANG



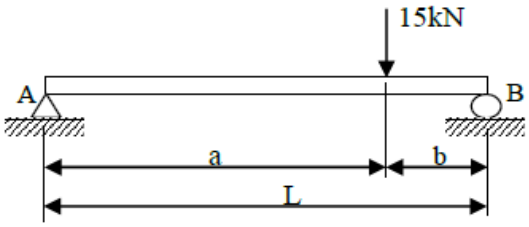


M = applied moment

Shear Forces and Bending Moments diagrams:

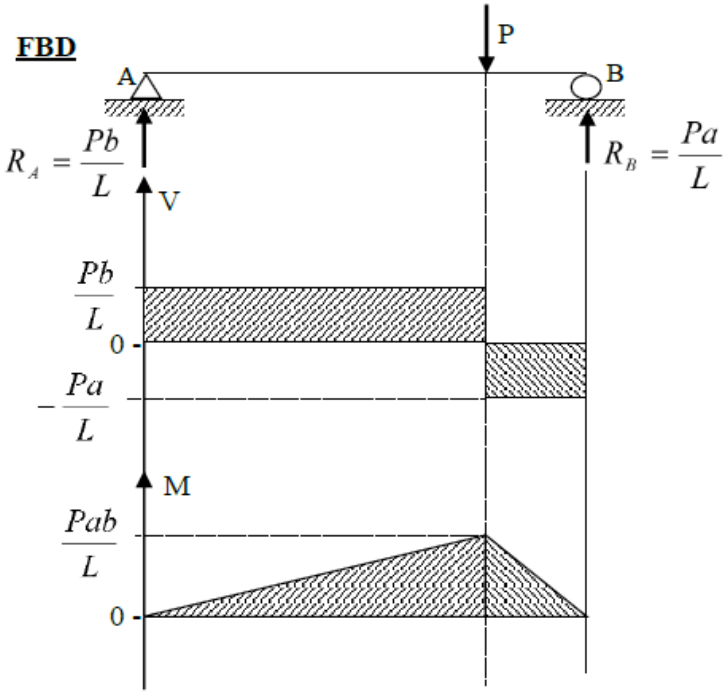
Example 1-9:

Skitch the shear force and bending moment diagrams for the following problem?



الحل:

نوجد ردود الأفعال عند الركائز



$$+\circlearrowleft \Sigma M_A = 0$$
$$R_B L - Pa = 0$$
$$R_B = \frac{Pa}{L}$$

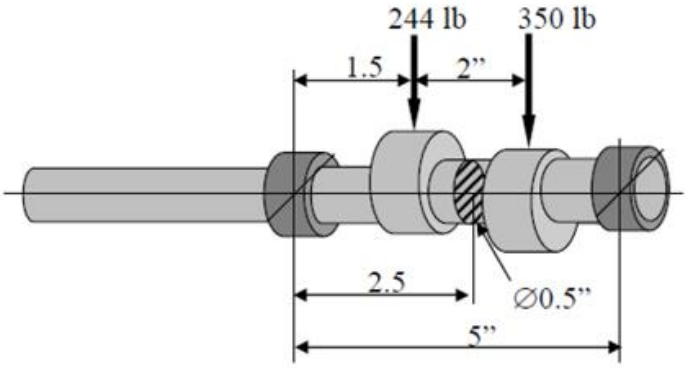
$$+\uparrow \Sigma F_y = 0$$
$$R_A + R_B = P$$

$$R_A = P - R_B = P - \frac{Pa}{L} = P\left(\frac{L-a}{L}\right)$$

$$R_A = \frac{Pb}{L}$$

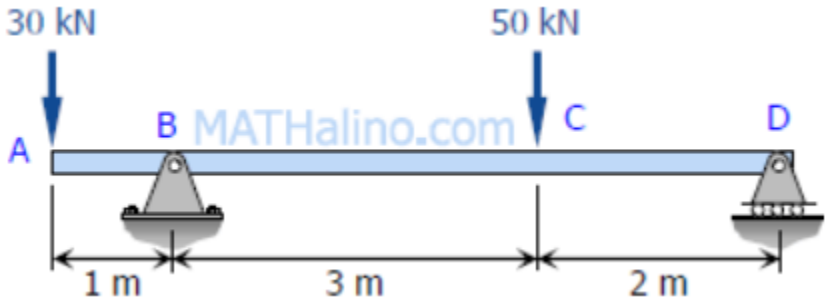
Example 1-10:

For the slider crank mechanism drive shaft shown, determine the bending stress at the indicated section?



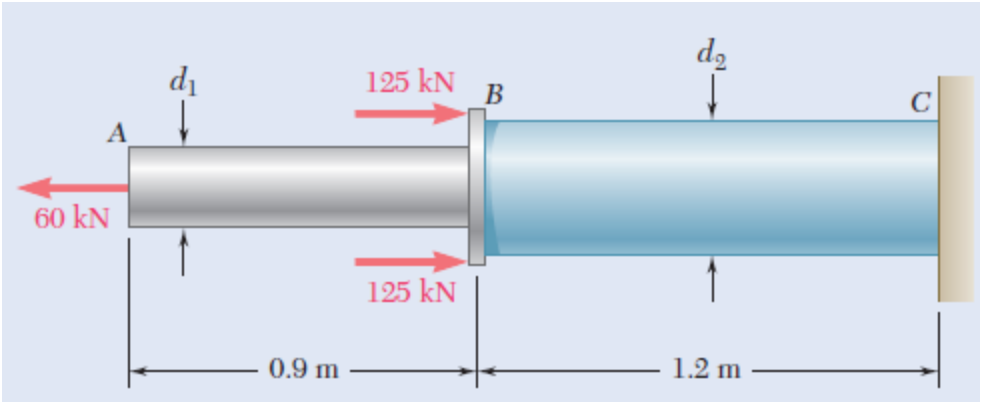
Example 1-11:

Draw shear and moment diagrams, specifying values at all change of loading positions and at points of zero shear. Neglect the mass of the beam in each problem.

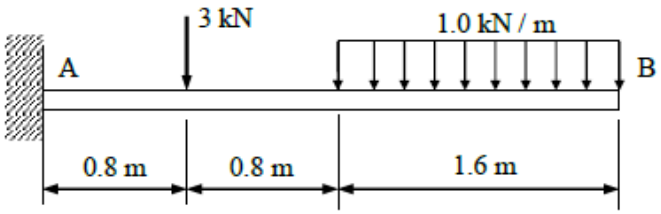


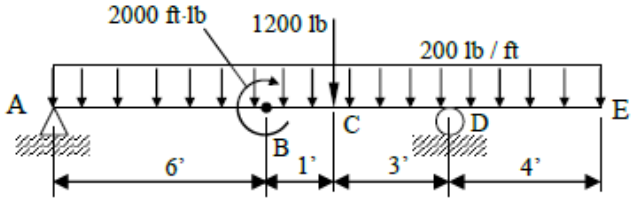
Example 1-12:

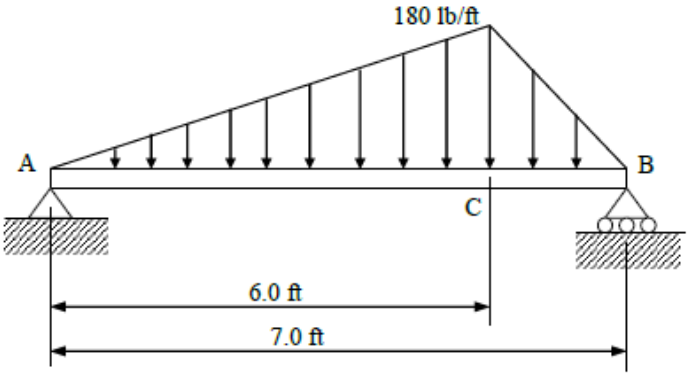
Two solid cylindrical rods *AB* and *BC* are welded together at *B* and loaded as shown. Knowing that *d*1 30 mm and *d*2 50 mm, find the average normal stress at the midsection of (*a*) rod *AB*, (*b*) rod *BC*.

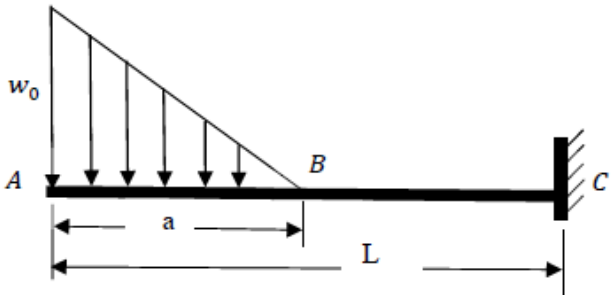


Extra Exercises:



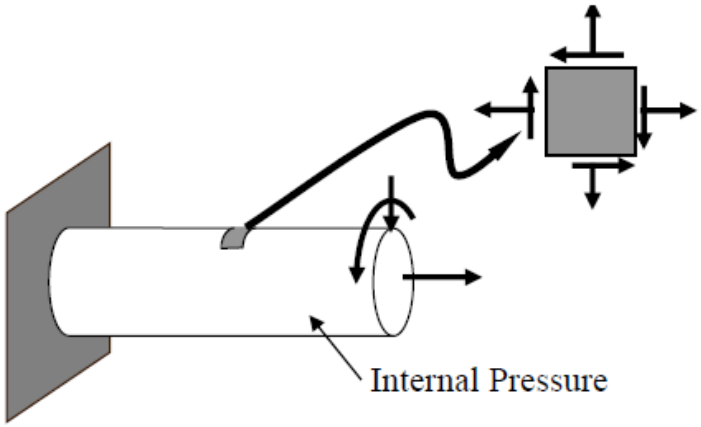




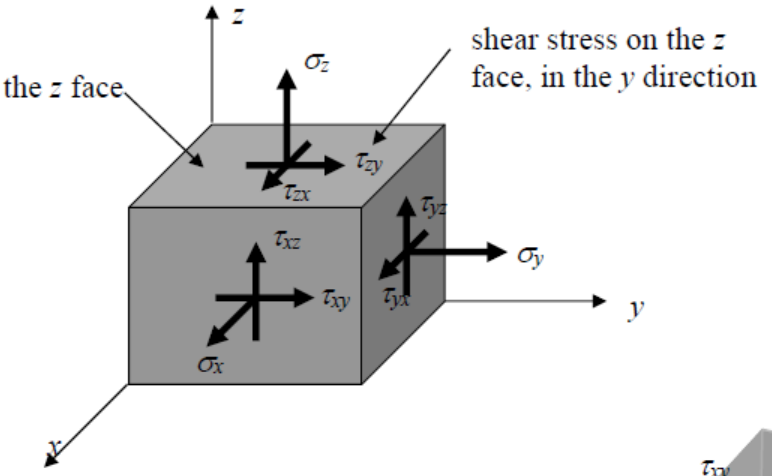


Combined Stresses: The Stress Element

Consider a point on a part that is under stress, and assign a coordinate system used to denote the stress values



In general, a cubic element at that point could be subjected to six stress values.

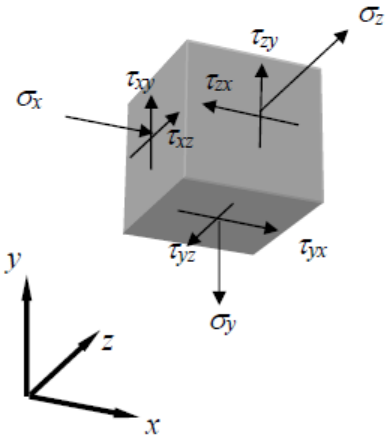


Notation for Stresses

τ_{xy}

Direction of stress component

Direction of the surface normal upon which the stress acts



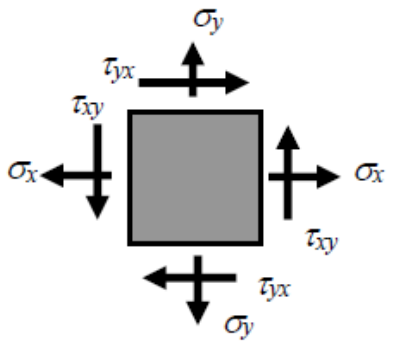
- Tensile normal stress are considered positive (arrow points away from face)
- Shear stresses are considered positive if the arrows on the positive face, are in the positive *x*-*y*-*z* direction.

Biaxial Stress

Consider the common case when one plane is unstressed.

$$\sigma_z = \tau_{xz} = \tau_{yz} = \tau_{zx} = \tau_{zy} = 0$$

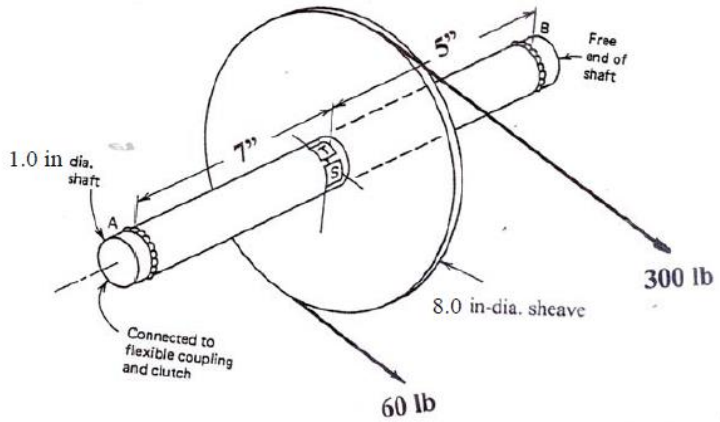
The remaining components are shown on a square stress element.



From equilibrium: $\sigma_x = \sigma_x$, $\tau_{xy} = \tau_{yx}$, $\tau_{xy} = \tau_{yx}$

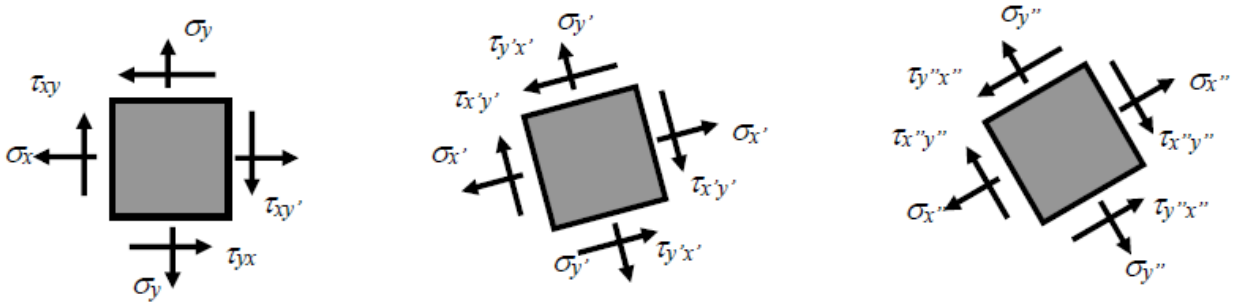
Example 2.6:

The shaft shown is mounted between self-aligning bearings A and B. The shaft can be modeled as a simply supported beam. Belt forces are applied to a sheave as shown. The left end of the shaft is connected to a motor by means of a flexible coupling. Nothing is attached to the right end. Elements T (top) and S (side) are located adjacent to the sheave. Determine (a) the torque applied on the shaft, (b) the maximum bending moment, (c) the maximum bending stress at the elements S and T, and (d) the maximum torsion stress on the elements S and T.



State of Stress

- By rotating the reference frame, the individual stress values change.
- The “state of stress” remains the same.



- Some configurations are more convenient than others

Principal Stresses

- Reference coordinates that yield extreme values of normal stress are of special interest.
- This is similar to determining the resultant of a force
- The maximum and minimum normal stresses are obtained through the conventional method to find extrema of a function. That is, determine θ such that

$$\frac{d\sigma_{x'}}{d\theta} = 0$$

- The angle, which produces the extreme values of normal stress, is

$$\theta_\sigma = \frac{1}{2}\tan^{-1}\left[\frac{2\tau_{xy}}{\sigma_x - \sigma_y}\right]$$

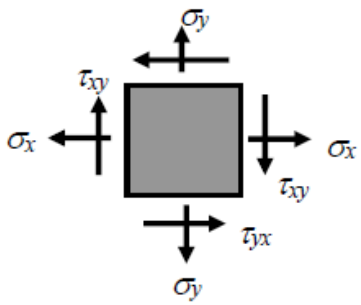
- The corresponding x' stress is called the maximum principle stress, σ_1

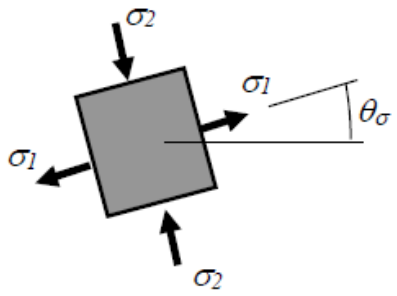
$$\sigma_{x'} = \sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^{\ 2}}$$

- The corresponding y' stress is called the minimum principle stress, σ_2

$$\sigma_{y'} = \sigma_2 = \frac{\sigma_x + \sigma_y}{2} - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^{\ 2}}$$

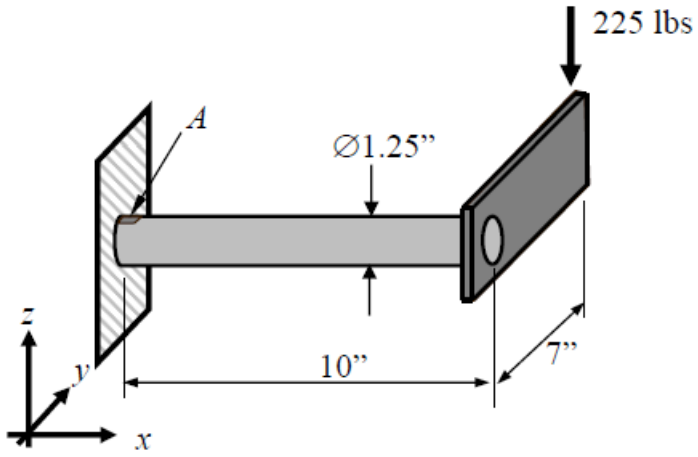
- The corresponding shear stress $\tau_{x'y'}$ is zero.
- After transformation to the maximum normal stress orientation, the stress element appears as





Example 2.7:

Calculate the principal normal stresses at point *A* of the shaft shown.


225 lbs
A
∅1.25”
z
y
x
10”
7”

Principle Shear Stress

- Likewise, the reference coordinates that yield extreme value of shear stress is of special interest.

- In this case, determine θ such that

$$\frac{d\tau_{x"y"}}{d\theta} = 0$$

- The angle, which produces the extreme values of the shear stress, is

$$\theta_\tau = \frac{1}{2}\tan^{-1}\left[-\frac{\sigma_x - \sigma_y}{2\tau_{xy}}\right]$$

- The corresponding shear stress is called the principal shear stress, τ_3.

$$\tau_{x"y"} = \tau_3 = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}{}^2}$$

- However, in the maximum shear orientation, the normal stresses are not zero.

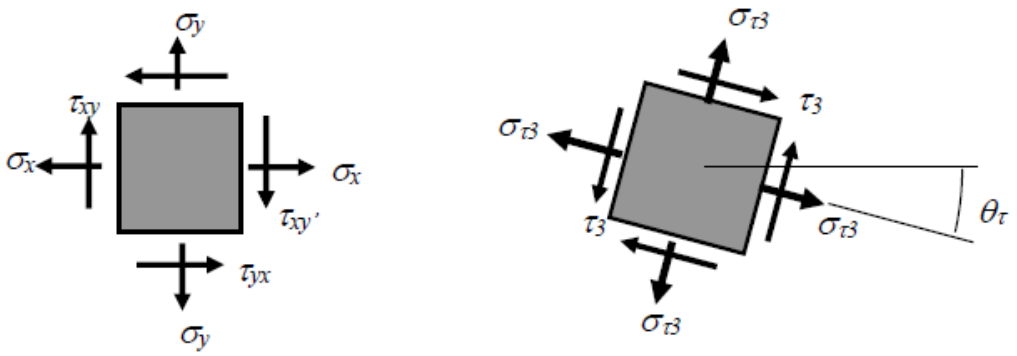
- The corresponding x' stress is

$$\sigma_{x"} = \sigma_{\tau 3} = \frac{\sigma_x + \sigma_y}{2}$$

- And the corresponding y' stress is also

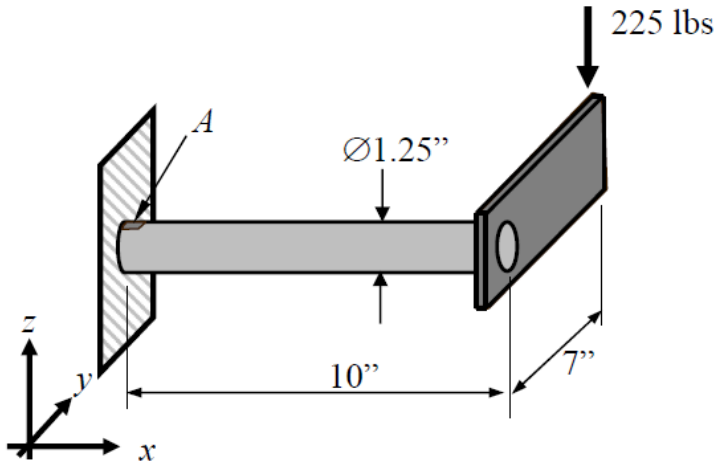
$$\sigma_{y"} = \sigma_{\tau 3} = \frac{\sigma_x + \sigma_y}{2}$$

- After transformation to the maximum shear stress orientation, the stress element appears as



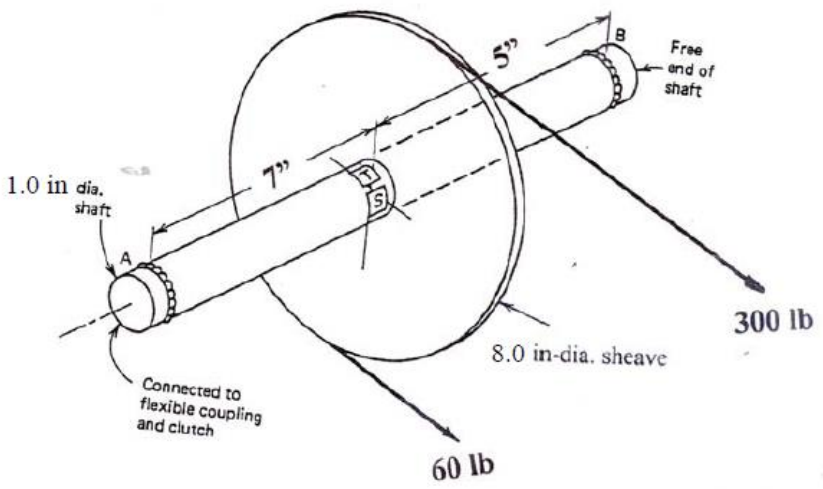
Example 2.8:

Calculate the principal shear stress at point *A* of the shaft shown?



Example 2.9

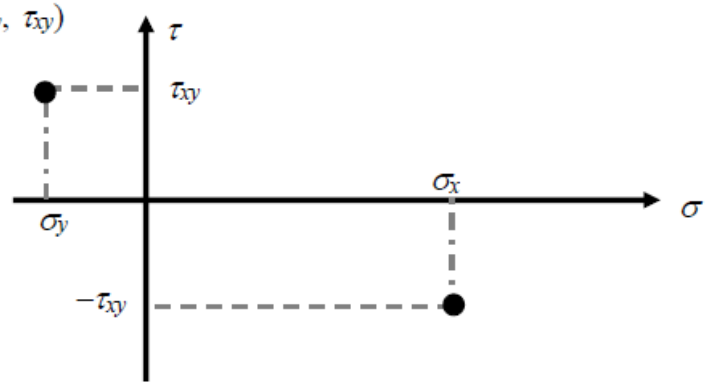
For the 1.0 in. diameter shaft shown. Determine the maximum principal shear stress at the elements *S* and *T*.



Mohr's Circle

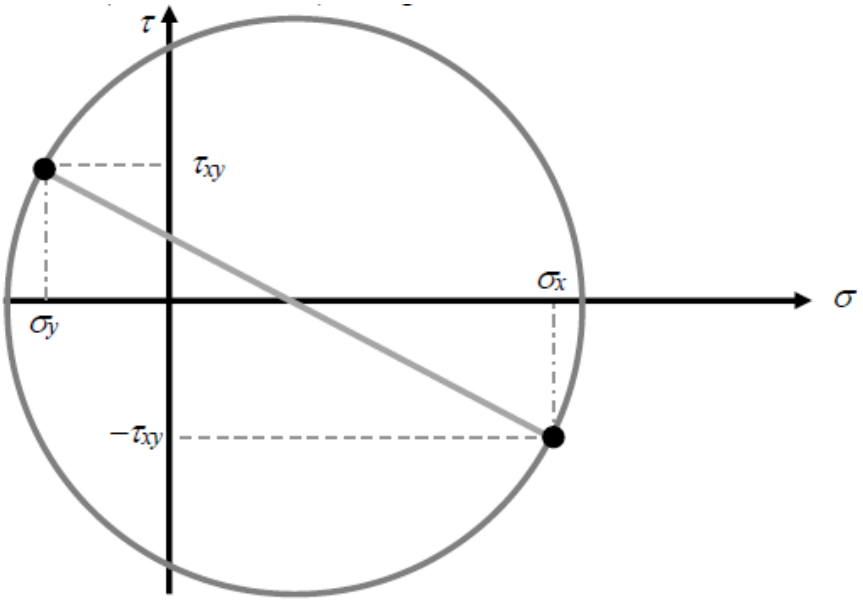
A graphical representation of the transformation equations

1. Create orthogonal σ and τ axes
2. Plot $(\sigma_x, -\tau_{xy})$ and (σ_y, τ_{xy})

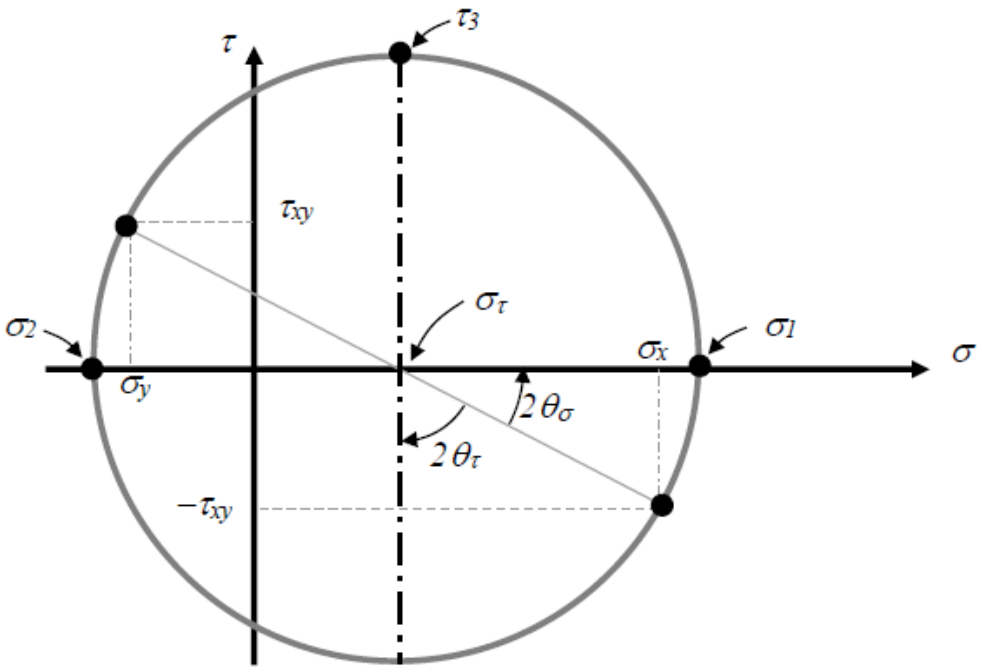


3. Create a straight line between the two points.

4. Construct a circle (*Mohr's Circle*) using the line as the as the diameter.

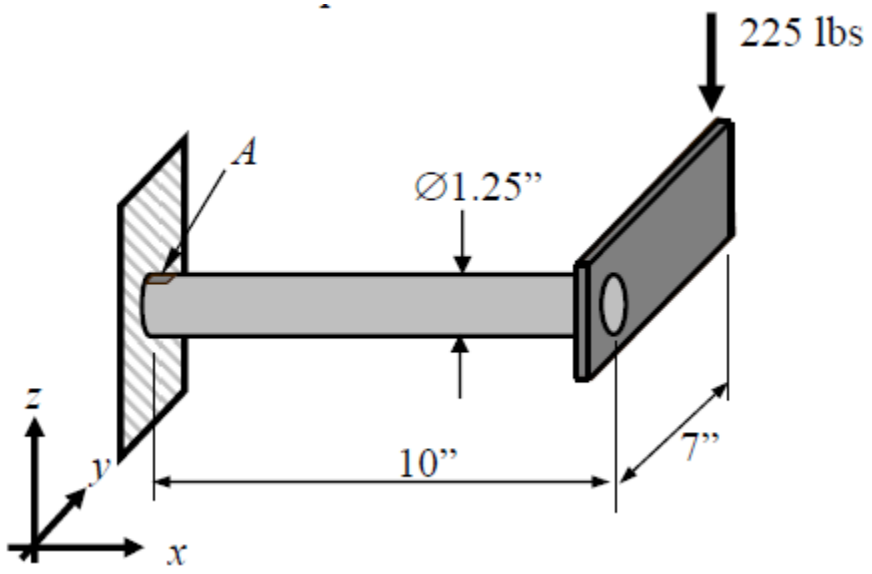


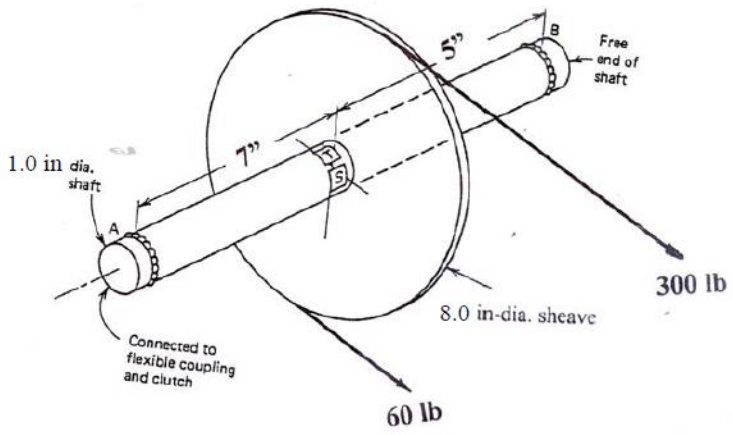
5. The key transformations can be visualized from Mohr's Circle



Example 2.10:

Construct Mohr's Circle for point *A* of the shaft shown?



Example 2-11:

Construct Mohr's circle for element T?